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Forecasting SARIMA Models in the Presence of an Seasonal Level Shift Outlier

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ABSTRACT
The ultimate objective of constructing time series models is to forecast future data accurately. However, the presence of outliers within the series can impact all phases of model development, inevitably affecting the accuracy of forecasts. This study seeks to examine how disregarding Seasonal Level Shift (SLS) outlier influences point forecasts generated by SARIMA models. Through analytical methods, we obtain the expression for the rise in the mean square error of h-step ahead forecasts attributable to the existence of an SLS outlier. To gain deeper insights into the findings, we conduct a simulation study. A key revelation is that SLS outlier notably elevate the mean square forecast error. Nevertheless, the extent of this escalation hinges not only on when the outlier appears relative to the forecast origin but also on its magnitude, number of years considered in the data, sample size, variance of errors and the parameter of the specific SARIMA model under consideration.

KEYWORDS
Time series, Forecasts, Seasonal level shift, Outlier, Mean square forecast error

1. Introduction

Outliers within time series arise from non-repetitive occurrences such as recording errors, significant economic or political shifts, the introduction of new regulations, and similar events. These outliers can induce various structural modifications to the time series. Four distinct types of outliers have been categorized based on their structural impacts in the literature. Initially, Fox (1972) defined two types of outliers: the Additive Outlier (AO) and the Innovational Outlier (IO). Subsequently, Chen and Tiao (1986) introduced the Level Shift (LS) outlier, and Tsay (1987) defined the Transient Change (TC) outlier. Extensive research has been conducted on outliers in time series context, one can refer to studies such as Hilmer (1984), Tsay (1988), Chen and Liu (1993), Chan (1995), Janhavi and Suresh (2011), and others. These works have reaveled that the presence of outliers significantly influences all stages of the Box-Jenkins approach to time series analysis, including identification, estimation, diagnostic checking, and forecasting.

A new type of seasonal outlier known as Seasonal Level Shift (SLS) was introduced by Kaiser and Maravall (2001), which is predominantly observed in seasonal time series. Subsequently Asghar and Urooj (2017) considered a modified seasonal level shift and analysed the performance of the test statistic in correct identification of the outlier.

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Recently, Shrivallabha and Suresh (2023) have studied the effect of the seasonal level shift, as defined by Kaiser and Maravall (2001), on the residuals of few seasonal models.

The main aim of building time series models is their utilization for forecasting, which offers invaluable insights facilitating decision-making processes and enabling organizations to devise long-term plans and strategies based on anticipated future trends. Nonetheless, the reliability of forecasts generated by these models becomes uncertain due to the disruptive impact of outliers throughout the modeling process. Ledolter (1989) examined the ramifications of disregarding additive outliers on both point forecasts and prediction intervals. Additionally, Trivez (1995) analyzed the inaccuracies in point forecasts and prediction intervals resulting from level shift and transient change outliers. More recently, Shahid et al.(2023) explored the impact of a 'modified' SLS outlier on forecast accuracy. Thus, an exploration of the impact of the SLS outlier, as proposed by Kaiser and Maravall (2001), on forecasts remains unexplored. It is noteworthy that this SLS outlier has been defined in the 'tsoutlier' R-software package, see López-de-Lacalle,J (2019), utilized for outlier detection in time series. This observation serves as a motivation for our work.

This study aims to examine the consequences of disregarding the SLS outlier on point forecasts from SARIMA models. Further, we explore the same for some specific seasonal models viz, Seasonal Autoregressive model of order one (SAR(1)), Seasonal Moving Average model of order one (SMA(1)), and Seasonal Autoregressive Moving Average model of order (1, 1) (SARMA(1, 1)), in terms of the mean square forecast error.

Let $\{X_t\}$ be generated by a seasonal autoregressive integrated moving average, $SARIMA(p,d,q)(P,D,Q)_s$ process given by

$$\Phi_P(B^s)\phi_p(B)(1-B)^d(1-B^s)^D X_t = \theta_q(B)\Theta_Q(B^s)a_t \tag{1}$$

$$X_t = \frac{\theta_q(B)\Theta_Q(B^s)}{\Phi_P(B^s)\phi_p(B)(1-B)^d(1-B^s)^D}a_t \tag{2}$$

where B is the backward shift operator such $B^a X_t = X_{t-a}$, $\phi(B) = 1 - \phi_1(B) - \phi_2(B^2) - \cdots - \phi_p(B^p), \Phi(B^s) = 1 - \Phi_1(B^s) - \Phi_2(B^{2s}) - \cdots - \phi_P(B^{Ps})$ and $\theta(B) = 1 - \theta_1(B) - \theta_2(B^2) - \cdots - \theta_q(B^q), \Theta(B^s) = 1 - \Theta_1(B^s) - \Theta_2(B^{2s}) - \cdots - \Theta_Q(B^{Qs})$ are polynomials with no common roots and all the roots lie outside the unit circle, d and D denotes the degree of nonseasonal and seasonal differencing respectively, $\{a_t\}$ follows i.i.d $\mathrm{N}(0,1)$ and s is the seasonal frequency. For simplicity, however, we shall assume that $\{X_t\}$ is a zero mean process. The model in (2) can also be written as

$$\left[\frac{\Phi_P(B^s)\phi_p(B)(1-B)^d(1-B^s)^D}{\theta_q(B)\Theta_Q(B^s)}\right] X_t = a_t$$

$$\pi(B)X_t = a_t \tag{3}$$

$$\implies \pi(B) = \frac{\Phi_P(B^s)\phi_p(B)(1-B)^d(1-B^s)^D}{\theta_q(B)\Theta_Q(B^s)}. \tag{4}$$

Next, the outlier model defined by Kaiser and Marvall (2001) for an observed time series perturbed by an SLS at time $t = T$ with magnitude δ is given by

$$Y_t = X_t + \frac{1}{1-B^s}\delta I_t^{(T)} \tag{5}$$

where Y_t is the observed, contaminated series, X_t is the unobserved, uncontaminated outlier free series, δ denotes the magnitude of the SLS outlier and $I_t^{(T)}$ is an indicator variable such that $I_t^{(T)} = 1$ if $t = T$ and 0 otherwise. Further, (5) can be written as

$$Y_t = \begin{cases} X_t & , t < T \\ X_t + \delta & , t = T, T+s, T+2s, ... \\ X_t & , \text{otherwise.} \end{cases} \tag{6}$$

The structure of the remaining paper is outlined as follows: Section 2 presents the derivation of the increase in mean square forecast error caused by the SLS outlier for SARIMA models and then focusing on specific models. Section 3 presents the numerical results. Finally, Section 4 provides concluding remarks.

2. Effect of SLS Outlier on Forecasts from SARIMA models with known coefficients

This section delineates the mathematical derivation of the impact of a seasonal level shift (SLS) outlier on forecasts generated by SARIMA models, expressed in terms of mean square forecast error. Subsequently, particular seasonal models are examined to gain deeper insights.

Assuming an idealistic but unrealistic scenario, let us consider that the coefficients of the SARIMA model and the time of occurrence of the outlier $t = T$ are known and sample size $n = ks$, where k is the number of years and s is the seasonal frequency. Suppose an outlier at time T is ignored and m is the number of observations prior to the forecast origin, then the h-step ahead prediction of the future observation Y_{T+m+h} from forecast origin $T + m$ ($\implies n = ks = T + m$) is given by

$$\begin{aligned} Y_{T+m}(h) &= \pi_1^{(h)} Y_{T+m} + \pi_2^{(h)} Y_{T+m-1} + \cdots \\ &= \sum_{j\geq 0} \pi_{j+1}^{(h)} Y_{T+m-j}, \end{aligned} \tag{7}$$

where the forecast weights are (see Box and Jenkins (2016; page 142))

$$\pi_j^{(h)} = \pi_{j+h-1} + \sum_{p=1}^{h-1} \pi_p \pi_j^{(h-p)}, j = 1, 2, \ldots, \tag{8}$$

and are calculated from $\pi_j^{(1)} = \pi_j$ weights using

$$\pi(B) = \frac{\Phi_P(B^s)\phi_p(B)\nabla_d \nabla_s^D}{\Theta_Q(B^s)\theta_q(B)}. \tag{9}$$

When $m < s$, using (6), the equation (7) becomes

$$Y_{T+m}(h) = \pi_1^{(h)} X_{T+m} + \pi_2^{(h)} X_{T+m-1} + \cdots + \pi_{m+1}^{(h)} \{X_T + \delta\} + \pi_{m+2}^{(h)} X_{T-1} + \cdots$$
$$Y_{T+m}(h) = \sum_{j\geq 0} \pi_{j+1}^{(h)} X_{T+m-j} + \pi_{m+1}^{(h)} \delta.$$

If $m = s$

$$\begin{aligned}
Y_{T+s}(h) &= \pi_1^{(h)} Y_{T+s} + \pi_2^{(h)} Y_{T+s-1} + \cdots + \pi_{s+1}^{(h)} Y_T + \pi_{s+2}^{(h)} Y_{T-1} + \cdots \\
&= \pi_1^{(h)} \{X_{T+s} + \delta\} + \pi_2^{(h)} X_{T+s-1} + \cdots + \pi_{s+1}^{(h)} \{X_T + \delta\} + \pi_{s+2}^{(h)} X_{T-1} + \cdots \\
Y_{T+s}(h) &= \sum_{j \geq 0} \pi_{j+1}^{(h)} X_{T+s-j} + \left\{ \pi_1^{(h)} + \pi_{s+1}^{(h)} \right\} \delta.
\end{aligned}$$

For the sake of simplicity, let us consider $T = \frac{n}{2} = \frac{ks}{2} \implies m = \frac{ks}{2}$. Hence (6) takes the form

$$Y_t = \begin{cases} X_t & , t < T \\ X_t + \delta & , t = T + js, j = 1, 2, \ldots, \frac{k}{2} \\ X_t & , \text{otherwise.} \end{cases} \tag{10}$$

Then using (10), $Y_{T+m}(h)$ can be written as

$$\begin{aligned}
Y_{T+m}(h) = \pi_1^{(h)} \{X_{T+(ks/2)} + \delta\} + \pi_2^{(h)} Y_{T+(ks/2)-1} + \cdots + \pi_{s+1}^{(h)} \{X_{T+(ks/2)-s} + \delta\} + \\
\pi_{s+2}^{(h)} Y_{T+(ks/2)-(s+1)} + \cdots + +\pi_{2s+1}^{(h)} \{X_{T+(ks/2)-2s} + \delta\} + \\
\pi_{2s+2}^{(h)} Y_{T+(ks/2)-(2s+1)} + \cdots + \pi_{(ks/2)+1}^{(h)} \{X_T + \delta\} + \\
\pi_{(ks/2)+2}^{(h)} Y_{T+(ks/2)-((ks/2)+1)} + \cdots .
\end{aligned}$$

Therefore

$$Y_{T+m}(h) = \sum_{j \geq 0} \pi_{j+1}^{(h)} X_{T+(ks/2)-j} + \left\{ \sum_{j=0}^{k/2} \pi_{js+1}^{(h)} \right\} \delta.$$

Subsequently, the h-step ahead forecast error

$$Y_{T+m+h} - Y_{T+m}(h) = e_{T+m}(h) - \left\{ \sum_{j=0}^{k/2} \pi_{js+1}^{(h)} \right\} \delta,$$

where $e_{T+m}(h) = a_{T+m+h} + \psi_1 a_{T+m+h-1} + \cdots + \psi_{h-1} a_{T+m+1}$ and $\psi_j, j = 1, 2, \ldots,$ are coefficients of B^j in $\psi(B) = \frac{\theta_q(B)\Theta_Q(B^s)}{\Phi_P(B^s)\phi_p(B)(1-B)^d(1-B^s)^D}$. Then the h-step ahead mean square forecast error is given by

$$MSFE(h; \delta, k, s) = \sigma_a^2 \sum_{j=0}^{h-1} \psi_j^2 + \left\{ \sum_{j=0}^{k/2} \pi_{js+1}^{(h)} \right\}^2 \delta^2. \tag{11}$$

The relative increase in the mean square forecast error (IMSFE) due to an outlier is found by

$$IMSFE(h; \delta, k, s) = \frac{MSFE(h; \delta, k, s) - MSFE(h; \delta = 0, k, s)}{MSFE(h; \delta = 0, k, s)}. \tag{12}$$

Hence using (11) in the above expression, the IMSFE due an SLS outlier turns out to be

$$IMSFE(h; \delta, k, s) = \left\{ \frac{\delta}{\sigma_a} \right\}^2 \frac{\left\{ \sum_{j=0}^{k/2} \pi_{js+1}^{(h)} \right\}^2}{\sum_{j=0}^{h-1} \psi_j^2}. \tag{13}$$

It is pertinant to note that the extent to which the forecasts are affected by ignoring the outlier of magnitude δ is determined by the forecast weights $\pi_{js+1}^{(h)}$.

To gain a clearer understanding of the aforementioned general result, we proceed to examine the increase in the mean square forecast error for SAR(1), SMA(1), and SARMA(1, 1) models, all being zero mean processes.

2.1. Increase in the Mean Sqaure Forecast Error in the SAR(1) Model

When $\pi(B) = 1 - \Phi B^s$ in (4), we get the SAR(1) model which is of the form

$$X_t = \Phi X_{t-s} + a_t.$$

In order to find the forecast weights, we start with

$$\pi(B) = 1 - \Phi B^s.$$

Expanding the left hand side of the above equation we get

$$1 - \pi_s B^s - \pi_2 s B^{2s} - \cdots = 1 - \Phi B^s.$$

Then upon solving we obtain

$$\pi_j = \begin{cases} \Phi & , j = s \\ 0 & , \text{otherwise}. \end{cases} \tag{14}$$

And

$$\psi(B) = \frac{1}{1 - \Phi B^s}$$
$$1 + \psi_s B^s + \psi_{2s} B^{2s} + \cdots = 1 + \Phi B^s + \Phi^2 B^{2s} + \cdots$$

$$\implies \psi_j = \begin{cases} 1 & , j = 0 \\ \Phi^i & , j = is, i = 1, 2, 3, \ldots \\ 0 & , \text{otherwise}. \end{cases} \tag{15}$$

Using (14) in (8), the forecast weights in (13) are found to be

$$\pi_{js+1}^{(h)} = \begin{cases} \Phi^i & , h = is, i = 1, 2, \ldots, j = 0 \\ 0 & , \text{otherwise}. \end{cases} \tag{16}$$

Therefore, after substituting (15) and (16) in (13), the relative increase in the mean square forecast error due to an SLS outlier in the case of SAR(1) model is

$$IMSFE(h;\delta,s)=\begin{cases}\left\{\dfrac{\delta}{\sigma_a}\right\}^2\dfrac{(\Phi^i)^2}{\sum_{v=1}^{i}(\Phi^{v-1})^2} & ,h=is,i=1,2,\ldots\\ 0 & ,\text{otherwise.}\end{cases}\tag{17}$$

Based on the equation above, we can deduce that the mean square forecast error increases solely for forecasts made at seasonal intervals. The magnitude of this increase depends on the outlier magnitude δ, error variance σ_a^2, and the model parameter Φ. Notably, it's crucial to recognize that this increase is unaffected by the number of years of data considered.

2.2. Increase in the Mean Sqaure Forecast Error in the SMA(1) Model

The SMA(1) model is obtained when, in (4) $\pi(B)=1-\Theta B^S$, therefore we get

$$X_t=a_t-\Theta a_{t-s}.$$

For the above model,

$$\pi(B)=\frac{1}{1-\Theta B^s}.$$

Expanding the equation above yields

$$1-\pi_s B^s-\pi_2 s B^{2s}-\cdots=1+\Theta B^s+\Theta^2 B^{2s}+\cdots.$$

Consequently, upon solving, we obtain

$$\pi_j=\begin{cases}1 & ,j=0\\ -\Theta^i & ,j=is,i=1,2,\ldots\\ 0 & ,\text{otherwise.}\end{cases}\tag{18}$$

And similarly

$$\psi(B)=1-\Theta B^s$$
$$1+\psi_s B^s+\psi_{2s}B^{2s}+\cdots=1-\Theta B^s$$

$$\implies \psi_j=\begin{cases}1 & ,j=0\\ -\Theta & ,j=s\\ 0 & ,\text{otherwise.}\end{cases}\tag{19}$$

Substituting (18) in(8), the forecast weights in (13) in the case of SMA(1) model are

$$\pi_{js+1}^{(h)}=\begin{cases}-\Theta^{j+1} & ,h=s,j=0,1,2,\ldots,\frac{k}{2}\\ 0 & ,\text{otherwise.}\end{cases}\tag{20}$$

Hence using (19) and (20) in (13), the relative increase in the mean square forecast error resulting from an SLS outlier in the SMA(1) model is

$$IMSFE(h;\delta,k,s)=\begin{cases}\left\{\dfrac{\delta}{\sigma_a}\right\}^2\left\{\sum_{r=1}^{\frac{k}{2}+1}\Theta^r\right\}^2 & ,h=s\\ 0 & ,\text{otherwise.}\end{cases} \tag{21}$$

The equation above indicates that for the SMA(1) model, an increase in the mean square forecast error solely occurs when the forecast horizon $h = s$. For nonseasonal and higher seasonal forecast horizons, the presence of an SLS outlier doesn't impact the forecasts. The increase in mean square forecast error due to the SLS outlier is determined by the model parameter Θ, the magnitude of the outlier δ, number of years of data k considered and the variance of the errors σ_a^2. Higher the value of k, higher will be the IMSFE.

2.3. Increase in the Mean Sqaure Forecast Error in SARMA(1, 1) Model

In equation (4), if $\pi(B)=\dfrac{1-\Phi B^S}{1-\Theta B^S}$, we obtain the SARMA(1, 1) expressed as

$$X_t-\Phi X_{t-s}=a_t-\Theta a_{t-s}.$$

For the above model,

$$\pi(B)=\frac{1-\Phi B^s}{1-\Theta B^s}.$$

Expanding the expression on both sides of the above equation yields

$$1-\pi_s B^s-\pi_2 sB^{2s}-\cdots=1+(\Theta-\Phi)B^s+\Theta(\Theta-\Phi)B^{2s}+\cdots.$$

Therefore, we get

$$\pi_j=\begin{cases}1 & ,j=0\\ \Theta^{i-1}(\Phi-\Theta) & ,j=is,i=1,2,\ldots\\ 0 & ,\text{otherwise.}\end{cases} \tag{22}$$

And

$$\psi(B)=\frac{1-\Theta B^s}{1-\Phi B^s}$$
$$1+\psi_s B^s+\psi_{2s}B^{2s}+\cdots=1+(\Phi-\Theta)B^s+\Phi(\Phi-\Theta)B^{2s}+\cdots$$

$$\implies \psi_j=\begin{cases}1 & ,j=0\\ \Phi^{i-1}(\Phi-\Theta) & ,j=is,i=1,2,\ldots\\ 0 & ,\text{otherwise.}\end{cases} \tag{23}$$

Plugging (22) and (23) into (8), the forecast weights in (13) for the SARMA(1, 1) model are obtained as follows

$$\pi_{js+1}^{(h)} = \begin{cases} \Phi^{i-1}\left(\Theta^{j}\right)(\Phi-\Theta) & , h = is, i = 1, 2, \ldots, j = 0, 1, 2, \ldots, \frac{k}{2} \\ 0 & , \text{otherwise.} \end{cases} \tag{24}$$

Hence, after subsituting (23) and (24) in (13), the IMSFE due to an SLS outlier in a series following SARMA(1, 1) model can be generalised as

$$IMSFE(h;\delta,k,s) = \begin{cases} \left\{\frac{\delta}{\sigma_a}\right\}^2 [(\Phi-\Theta)]^2 \left\{\sum_{r=0}^{\frac{k}{2}} \Theta^r\right\}^2 & , h = s \\ \dfrac{\left\{\frac{\delta}{\sigma_a}\right\}^2 \left[\Phi^{i-1}(\Phi-\Theta)\right]^2 \left\{\sum_{r=0}^{\frac{k}{2}} \Theta^r\right\}^2}{1+\sum_{j=1}^{i-1}(\Phi^{j-1}(\Phi-\Theta))^2} & , h = is, i = 2, 3, \ldots \\ 0 & , \text{otherwise.} \end{cases} \tag{25}$$

It can be inferred from the above equation that the presence of the SLS outlier influences forecasts solely when the forecast horizon h aligns with a multiple of s. As the value of h increases, the IMSFE diminishes gradually. The IMSFE attributed to the SLS outlier in the SARMA(1, 1) model is dependent on the parameters Φ and Θ, the magnitude of the outlier δ, number of years of data k considered, and the variance of errors σ_a^2. The IMSFE increases as k increases.

3. Numerical Results

The percent IMSFE is calculated using equations (17), (21), and (25) for SAR(1), SMA(1), and SARMA(1, 1) models, respectively. This percent IMSFE is evaluated across various parameter values, sample sizes $n = 120, 180$, forecast horizons $h = s, 2s, 3s$, and outlier magnitudes $\delta = 3\sigma_a, 5\sigma_a, 10\sigma_a$. The results for SAR(1), SMA(1), and SARMA(1, 1) models are tabulated in Tables A1, A2, and A3 - A8 respectively. Additionally, Figures B1, B2, and B3 - B4 illustrate findings related to SAR(1), SMA(1), and SARMA(1, 1) models, respectively. All the tables and figures have been provided in the appendix.

The unprecedented COVID-19 epidemic has put the world in peril and shifted the global landscape in unanticipated ways. COVID-19's entry into global space has resulted in a public health emergency as well as an economic crisis. Global and national health systems have been preoccupied with the virus's treatment, containment, and vaccine development as a public health emergency. Furthermore, the government's global lockdown to stop the virus from spreading has triggered an economic catastrophe due to supply and demand shocks. Thus, the labour market, global supply chains, consumer consumption, and stock market are all important routes through which the lockdown will impact the global economy. The Nigeria stock exchange and its volatility are key factors that influence economic and financial activities in Nigeria that is why stock exchange market fluctuation have always attracted favorable recognition in both economic, financial and statistics literature. The need for modelling and forecasting volatility is because investors are not only interested in the average returns of a stock but also its risk. Therefore, market investors and speculators need information to analyze the profit or loss for the erratic behaviour of financial asset. However, analyzing volatility is helpful as it informs investors a measure of the risk involved in holding an asset The aim of this work is to model the impact of Post COVID -19 on the Nigeria stock exchange using GARCH, Exponential GARCH and GJR- GARCH Models while the objective is to forecast the volatility of the Nigeria stock exchange. Economic impacts of epidemics and pandemics have been examined in literature in forms of country-specific and global studies. However, COVID-19 has created an unprecedented global disruption which has necessitated several studies. Consequently, emphasis in this review will be on macroeconomic and financial market disruptions orchestrated by COVID-19 pandemic. (Oyelami et al.,2020) in their study investigates the dynamic interaction of COVID-19 incidence and stock market performance using daily time series data between April, 2020 and August, 2020 of All Share Index (ASI), COVID-19 pandemic confirmed cases, Nigeria's borrowing rate and exchange rate. It spans through the pre-lockdown, lockdown and post lockdown periods. Based on the assumption of endogeneity, vector autoregressive (VAR) model was employed for estimation. The result revealed that COVID19 confirmed cases have a significant negative effect on stock market performance proxy by stock market returns. (Adenomonet al.,2020) employed EGARCH and QGARCH models with addition of dummy variable to allow for non COVID-19 and COVID-19 period in their study on the effects of COVID-19 outbreak on the Nigerian Stock Exchange performance: evidence from GARCH Model they discovered that EGARCH (1,1) with SSTD by incorporating the COVID-19 period emerged the best model among the competing models. The result revealed a negative impact of COVID-19 on the stock returns in Nigeria under the period under study. (Egunjobi, 2022) in his study explained that the financial market and consequently the economic climate have been severely impacted by the economic chaos caused by the pandemic. The consequences are reflected in the inability of the financial sector to perform its function of promoting development via income generation and reducing poverty and inequality. Even while stock market returns have been hurt more severely, especially in service delivery and reduction in turnover, the business environment is still very uncertain and unpredictable, though the study revealed that this has not really deterred investors or operations in the Nigerian financial sector. Thus, to achieve economic development, a sustainability appropriate policies and relief measures must be geared towards reducing the negative consequences arising from the pandemic. (Arashiet al., 2022) observed in their study that with the continuous development of economy in the society, a rapid rise has happened in emergence of capital markets in the world today. They concluded that investing in stock market forms an important part of the economy of the society. They modelled daily return series of stock index NASDAQ stock exchange using ARMA-GARCH model. (Olayemi et al., 2022) reflected three different models in their empirical work. They modelled the volatility in Nigeria crude oil price using the symmetric and asymmetric GARCH model that capture most common stylized facts about Crude oil price in Nigeria markets such as volatility clustering and leverage effects. It was discovered that GARCH (1,1) model outperformed EGARCH (1,1) and PGARCH (1,1) models because it has the least Akaike info Criterion (AIC)" Alzyadat& Asfoura (2021) observed that the descriptive statistics show that stock market returns and the number of COVID-19 infection cases recorded during the study period are volatile, and displays evidence of fluctuations in the variables over the study period.Based on the results of the impulse response functions that shock market returns respond to COVID-19 negatively over the study period.

4. Numerical Results

The percent IMSFE is calculated using equations (17), (21), and (25) for SAR(1), SMA(1), and SARMA(1, 1) models, respectively. This percent IMSFE is evaluated across various parameter values, sample sizes $n = 120, 180$, forecast horizons $h = s, 2s, 3s$, and outlier magnitudes $\delta = 3\sigma_a, 5\sigma_a, 10\sigma_a$. The results for SAR(1), SMA(1), and SARMA(1, 1) models are tabulated in Tables A1, A2, and A3 - A8 respectively. Additionally, Figures B1, B2, and B3 - B4 illustrate findings related to SAR(1), SMA(1), and SARMA(1, 1) models, respectively. All the tables and figures have been provided in the appendix.

From tables (A1 - A8) and figures (B1 - B4), several observations emerge: Regardless of the model, the percent IMSFE rises as δ grows, while the percent IMSFE falls as the prediction horizon h increases. For both SMA(1) and SARMA(1, 1) models, as n increases there is a corresponding rise in the percent IMSFE. Under the SAR(1) model, the percent IMSFE increases as $|\Phi| \uparrow 1$. Further, the percent IMSFE remains identical for both positive and negative values of the model parameter Φ. In the case of the SMA(1) model, the percent IMSFE grows at a higher rate as Θ approaches 1, while the minimum is noted when Θ gets closer to -1. As for the SARMA(1,1) model, the percent IMSFE peaks when $\Phi < -0.5$ and $\Theta > 0.5$, compared to other permissible parameter ranges.

5. Conclusion

The hazardous effect time series outliers can have on forecasting is well documented in the literature and our present work also points out the same issue in the case of an SLS outlier. The presence of SLS outlier in the series results in an increase in the mean square forecast error but only when the forecasts are made for seasonal horizons. This aspect stands out as a distinctive feature of our study when contrasted with Ledolter (1989) and Trivez (1995). Comparing with Shahid (2023), the increase in mean square forecast error caused by the SLS outlier is more pronounced than that caused by the modified SLS outlier. In our study the increase in the mean square forecast error due to the SLS outlier for SAR(1), SMA(1) and SARMA(1, 1) models were derived. From (17), (21) and (25), we can conclude that the increase is dependent on the magnitude of the outlier δ, variance of the errors σ_a^2, parameter (s) of the model considered and the number of the years k considered in the data. Therefore, one must identify and eliminate the SLS outlier if present in the series otherwise the forecast accuracy will be compromised. While our present work focuses on a single SLS outlier, it can be extended to encompass multiple outlier scenarios, involving either the same type of outlier or a combination of different types. Our study also has the scope for extension to other seasonal models and explore outlier incidences at time points beyond $T = \frac{n}{2}$. The present work is carried out under the assumption that the parameter values are known, which can also be relaxed.

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6. Appendices

Appendix A. Tables

	$n = 120$								
Oultier magnitude $\rightarrow$	$\delta = 3\sigma_a$			$\delta = 5\sigma_a$			$\delta = 10\sigma_a$		
$\Phi \downarrow$ $h \rightarrow$	s	$2s$	$3s$	s	$2s$	$3s$	s	$2s$	$3s$
-0.9	729.000	326.238	193.949	2025.000	906.215	538.746	8100.000	3624.862	2154.986
-0.8	576.000	224.780	115.110	1600.000	624.390	319.750	6400.000	2497.561	1279.001
-0.7	441.000	145.027	61.201	1225.000	402.852	170.003	4900.000	1611.409	680.013
-0.6	324.000	85.765	28.189	900.000	238.235	78.303	3600.000	952.941	313.212
-0.5	225.000	45.000	10.714	625.000	125.000	29.762	2500.000	500.000	119.048
-0.4	144.000	19.862	3.109	400.000	55.172	8.637	1600.000	220.690	34.548
-0.3	81.000	6.688	0.597	225.000	18.578	1.660	900.000	74.312	6.639
-0.2	36.000	1.385	0.055	100.000	3.846	0.154	400.000	15.385	0.614
-0.1	9.000	0.089	0.001	25.000	0.248	0.002	100.000	0.990	0.010
0.1	9.000	0.089	0.001	25.000	0.248	0.002	100.000	0.990	0.010
0.2	36.000	1.385	0.055	100.000	3.846	0.154	400.000	15.385	0.614
0.3	81.000	6.688	0.597	225.000	18.578	1.660	900.000	74.312	6.639
0.4	144.000	19.862	3.109	400.000	55.172	8.637	1600.000	220.690	34.548
0.5	225.000	45.000	10.714	625.000	125.000	29.762	2500.000	500.000	119.048
0.6	324.000	85.765	28.189	900.000	238.235	78.303	3600.000	952.941	313.212
0.7	441.000	145.027	61.201	1225.000	402.852	170.003	4900.000	1611.409	680.013
0.8	576.000	224.780	115.110	1600.000	624.390	319.750	6400.000	2497.561	1279.001
0.9	729.000	326.238	193.949	2025.000	906.215	538.746	8100.000	3624.862	2154.986

Table A1.: Percent IMSFE due to an SLS outlier in the SAR(1) model

	$n = 120$			$n = 180$		
Oultier magnitude $\rightarrow$	$\delta = 3\sigma_a$	$\delta = 5\sigma_a$	$\delta = 10\sigma_a$	$\delta = 3\sigma_a$	$\delta = 5\sigma_a$	$\delta = 10\sigma_a$
$\Theta \downarrow$ $h \rightarrow$	s	s	s	s	s	s
-0.9	44.335	123.153	492.614	65.502	181.951	727.805
-0.8	96.788	268.855	1075.42	123.129	342.026	1368.105
-0.7	118.802	330.005	1320.021	135.509	376.413	1505.652
-0.6	115.028	319.523	1278.091	122.347	339.852	1359.408
-0.5	96.899	269.165	1076.66	99.22	275.612	1102.448
-0.4	72.869	202.413	809.653	73.373	203.814	815.257
-0.3	47.859	132.942	531.768	47.923	133.119	532.475
-0.2	24.997	69.436	277.742	25.000	69.444	277.776
-0.1	7.438	20.661	82.644	7.438	20.661	82.645
0.1	11.111	30.864	123.457	11.111	30.864	123.457
0.2	56.243	156.23	624.92	56.25	156.249	624.997
0.3	165.065	458.514	1834.058	165.284	459.123	1836.494
0.4	396.73	1102.028	4408.11	399.476	1109.655	4438.621
0.5	872.095	2422.485	9689.941	892.982	2480.507	9922.028
0.6	1840.451	5112.364	20449.458	1957.547	5437.63	21750.52
0.7	3814.862	10596.839	42387.357	4351.334	12087.038	48348.152
0.8	7839.813	21777.259	87109.036	9973.486	27704.127	110816.508
0.9	16005.015	44458.376	177833.505	23646.398	65684.439	262737.755

Table A2.: Percent IMSFE due to an SLS outlier in the SMA(1) model

Θ↓ Φ→	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
									$h = s$										
-0.8	1.512	0.000	1.512	6.049	13.611	24.197	37.808	54.443	74.103	96.788	122.497	151.231	182.989	217.773	255.580	296.413	340.270	387.151	437.057
-0.7	9.698	2.425	0.000	2.425	9.698	21.821	38.792	60.613	87.283	118.802	155.170	196.387	242.453	293.368	349.132	409.745	475.208	545.519	620.679
-0.6	28.757	12.781	3.195	0.000	3.195	12.781	28.757	51.124	79.881	115.028	156.566	204.495	258.813	319.523	386.623	460.113	539.993	626.265	718.926
-0.5	62.016	34.884	15.504	3.876	0.000	3.876	15.504	34.884	62.016	96.899	139.535	189.923	248.063	313.954	387.598	468.993	558.141	655.040	759.691
-0.4	113.857	72.869	40.989	18.217	4.554	0.000	4.554	18.217	40.989	72.869	113.857	163.955	223.161	291.475	368.898	455.430	551.070	655.819	769.676
-0.3	191.437	132.942	85.083	47.859	21.271	5.318	0.000	5.318	21.271	47.859	85.083	132.942	191.437	260.566	340.332	430.732	531.768	643.440	765.746
-0.2	306.211	224.971	156.230	99.987	56.243	24.997	6.249	0.000	6.249	24.997	56.243	99.987	156.230	224.971	306.211	399.949	506.185	624.920	756.153
0	476.032	364.462	267.768	185.950	119.008	66.942	29.752	7.438	0.000	7.438	29.752	66.942	119.008	185.950	267.768	364.462	476.032	602.478	743.800
-0.1	729.000	576.000	441.000	324.000	225.000	144.000	81.000	36.000	9.000	0.000	9.000	36.000	81.000	144.000	225.000	324.000	441.000	576.000	729.000
0.1	1111.109	899.998	711.110	544.443	399.999	277.777	177.777	100.000	44.444	11.111	0.000	11.111	44.444	100.000	177.777	277.777	399.999	544.443	711.110
0.2	1701.345	1406.070	1138.917	899.885	688.974	506.185	351.518	224.971	126.546	56.243	14.061	0.000	14.061	56.243	126.546	224.971	351.518	506.185	688.974
0.3	2641.043	2219.210	1834.058	1485.587	1173.797	898.688	660.261	458.514	293.449	165.065	73.362	18.341	0.000	18.341	73.362	165.065	293.449	458.514	660.261
0.4	4190.460	3570.569	3000.270	2479.562	2008.445	1586.920	1214.985	892.642	619.890	396.730	223.161	99.182	24.796	0.000	24.796	99.182	223.161	396.730	619.890
0.5	6837.223	5895.360	5023.266	4220.938	3488.379	2825.587	2232.563	1709.306	1255.816	872.095	558.141	313.954	139.535	34.884	0.000	34.884	139.535	313.954	558.141
0.6	11502.820	10020.234	8639.896	7361.805	6185.961	5112.364	4141.015	3271.913	2505.059	1840.451	1278.091	817.978	460.113	204.495	51.124	0.000	51.124	204.495	460.113
0.7	19930.708	17517.224	15259.448	13157.382	11211.023	9420.374	7785.433	6306.201	4982.677	3814.862	2802.756	1946.358	1245.669	700.689	311.417	77.854	0.000	77.854	311.417
0.8	35401.657	31359.253	27561.844	24009.428	20702.007	17639.580	14822.147	12249.708	9922.264	7839.813	6002.357	4409.895	3062.427	1959.953	1102.474	489.988	122.497	0.000	122.497
									$h = 2s$										
-0.8	1.213	0.000	0.734	2.094	3.122	3.338	2.722	1.601	0.497	0.000	0.677	3.025	7.452	14.280	23.753	36.050	51.302	69.600	91.007
-0.7	7.553	1.536	0.000	0.864	2.331	3.203	3.010	1.940	0.642	0.000	0.946	4.340	10.910	21.239	35.772	54.836	78.666	107.425	141.222
-0.6	21.370	7.865	1.550	0.000	0.791	1.966	2.374	1.763	0.639	0.000	1.051	4.988	12.869	25.562	43.736	67.885	98.363	135.409	179.179
-0.5	43.304	20.482	7.305	1.382	0.000	0.614	1.342	1.280	0.535	0.000	1.026	5.099	13.613	27.753	48.450	76.397	112.086	155.846	207.889
-0.4	73.780	40.203	18.426	6.306	1.127	0.000	0.406	0.701	0.376	0.000	0.911	4.822	13.479	28.437	50.953	81.977	122.183	172.018	231.761
-0.3	114.017	68.066	35.940	15.807	5.113	0.842	0.000	0.211	0.205	0.000	0.733	4.254	12.669	27.980	51.880	85.671	130.283	186.335	254.203
-0.2	166.464	105.869	61.242	31.031	12.900	3.846	0.557	0.000	0.062	0.000	0.516	3.448	11.249	26.467	51.378	87.794	137.034	199.974	277.142
-0.1	235.113	156.547	96.475	53.554	25.648	9.826	2.575	0.295	0.000	0.000	0.286	2.457	9.233	23.802	49.222	88.058	142.229	213.031	301.239
0	326.238	224.780	145.027	85.765	45.000	19.862	6.688	1.385	0.089	0.000	0.089	1.385	6.688	19.862	45.000	85.765	145.027	224.780	326.238
0.1	449.999	318.231	212.466	131.543	73.529	35.555	13.793	3.670	0.427	0.000	0.000	0.440	3.846	14.679	38.314	80.000	144.117	233.855	351.219
0.2	623.570	449.942	308.326	197.536	115.600	59.551	25.309	7.758	1.161	0.000	0.139	0.000	1.253	8.653	29.024	69.819	137.795	238.205	374.543
0.3	876.740	642.667	449.344	295.476	178.932	96.503	43.694	14.672	2.530	0.000	0.705	0.726	0.000	2.905	17.635	54.517	123.957	234.759	393.244
0.4	1261.811	936.543	665.218	446.321	277.410	154.821	73.388	26.254	4.959	0.000	2.047	3.815	2.210	0.000	6.138	34.332	100.320	218.885	401.689
0.5	1870.997	1402.614	1008.771	687.574	436.047	249.776	122.519	45.887	9.234	0.000	4.812	11.521	12.075	5.526	0.000	12.434	65.743	184.340	389.736
0.6	2866.857	2166.537	1573.810	1086.168	699.769	408.989	205.907	79.803	16.812	0.000	10.225	28.206	37.991	31.461	12.654	0.000	24.803	125.843	341.919
0.7	4534.796	3449.546	2526.057	1760.839	1148.670	682.018	350.344	139.364	30.382	0.000	20.608	62.283	96.647	102.853	74.860	27.750	0.000	49.333	242.546
0.8	7371.553	5637.619	4155.478	2920.066	1923.978	1156.694	603.617	244.994	54.819	0.000	40.284	129.703	220.495	270.338	252.861	169.611	59.429	0.000	98.240
									$h = 3s$										
-0.8	0.975	0.000	0.358	0.744	0.765	0.522	0.241	0.063	0.005	0.000	0.007	0.119	0.639	2.088	5.132	10.480	18.771	30.505	46.021
-0.7	5.933	0.977	0.000	0.310	0.577	0.506	0.268	0.077	0.006	0.000	0.009	0.171	0.940	3.125	7.793	16.100	29.104	47.643	72.286
-0.6	16.225	4.913	0.756	0.000	0.197	0.313	0.212	0.070	0.006	0.000	0.010	0.196	1.113	3.787	9.617	20.156	36.853	60.867	92.989
-0.5	31.551	12.451	3.513	0.496	0.000	0.098	0.120	0.051	0.005	0.000	0.010	0.201	1.184	4.144	10.767	22.975	42.602	71.138	109.604
-0.4	51.430	23.643	8.678	2.239	0.281	0.000	0.036	0.028	0.004	0.000	0.009	0.191	1.178	4.282	11.456	25.010	47.205	79.909	124.414
-0.3	76.048	38.619	16.496	5.526	1.266	0.135	0.000	0.008	0.002	0.000	0.007	0.169	1.114	4.253	11.817	26.562	51.276	88.310	139.310
-0.2	106.474	57.940	27.330	10.643	3.160	0.612	0.050	0.000	0.001	0.000	0.005	0.137	0.994	4.063	11.869	27.712	55.070	96.957	155.516
-0.1	144.702	82.770	41.845	17.984	6.198	1.552	0.231	0.012	0.000	0.000	0.003	0.098	0.821	3.690	11.542	28.345	58.505	105.985	173.668
0	193.949	115.110	61.201	28.189	10.714	3.109	0.597	0.055	0.001	0.000	0.001	0.055	0.597	3.109	10.714	28.189	61.201	115.110	193.949
0.1	259.430	158.323	87.396	42.343	17.241	5.512	1.226	0.146	0.004	0.000	0.000	0.018	0.345	2.318	9.259	26.866	62.510	123.644	216.160
0.2	349.911	218.154	123.909	62.353	26.704	9.141	2.238	0.309	0.012	0.000	0.001	0.000	0.113	1.376	7.109	23.946	61.493	130.366	239.565
0.3	480.476	304.580	176.850	91.612	40.757	14.669	3.841	0.582	0.025	0.000	0.007	0.029	0.000	0.464	4.367	19.060	56.894	133.197	262.289
0.4	677.366	435.062	257.007	136.166	62.374	23.316	6.415	1.039	0.049	0.000	0.020	0.152	0.199	0.000	1.531	12.191	47.245	128.724	280.007
0.5	986.433	640.243	383.420	206.771	96.899	37.294	10.653	1.812	0.092	0.000	0.048	0.459	1.083	0.883	0.000	4.460	31.618	112.056	283.961
0.6	1487.827	973.872	589.648	322.502	153.880	60.591	17.814	3.143	0.168	0.000	0.102	1.122	3.394	5.003	3.156	0.000	12.095	78.605	259.592
0.7	2321.169	1529.862	934.546	516.977	250.246	100.333	30.173	5.477	0.303	0.000	0.206	2.472	8.592	16.242	18.537	9.955	0.000	31.375	190.527
0.8	3727.716	2470.905	1520.413	848.871	415.703	169.103	51.774	9.608	0.546	0.000	0.402	5.134	19.494	42.320	61.937	60.226	28.980	0.000	78.941

Table A3.: Percent IMSFE due to an SLS outlier in the SARMA(1, 1) model($n = 120, \delta = 3\sigma_a$)-a

Θ↓ Φ→	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
									$h = s$										
-0.8	4.201	0.000	4.201	16.803	37.808	67.214	105.022	151.231	205.842	268.855	340.270	420.086	508.304	604.924	709.945	823.369	945.194	1075.420	1214.049
-0.7	26.939	6.735	0.000	6.735	26.939	60.613	107.757	168.370	242.453	330.005	431.027	545.519	673.480	814.911	969.812	1138.182	1320.021	1515.331	1724.110
-0.6	79.881	35.503	8.876	0.000	8.876	35.503	79.881	142.010	221.891	319.523	434.906	568.040	718.926	887.563	1073.952	1278.091	1499.982	1739.624	1997.017
-0.5	172.266	96.899	43.066	10.767	0.000	10.767	43.066	96.899	172.266	269.165	387.598	527.563	689.063	872.095	1076.660	1302.759	1550.391	1819.556	2110.254
-0.4	316.271	202.413	113.857	50.603	12.651	0.000	12.651	50.603	113.857	202.413	316.271	455.430	619.890	809.653	1024.717	1265.083	1530.750	1821.719	2137.990
-0.3	531.768	369.283	236.341	132.942	59.085	14.771	0.000	14.771	59.085	132.942	236.341	369.283	531.768	723.796	945.366	1196.478	1477.134	1787.332	2127.073
-0.2	850.586	624.920	433.972	277.742	156.230	69.436	17.359	0.000	17.359	69.436	156.230	277.742	433.972	624.920	850.586	1110.969	1406.070	1735.889	2100.426
-0.1	1322.311	1012.395	743.800	516.528	330.578	185.950	82.644	20.661	0.000	20.661	82.644	185.950	330.578	516.528	743.800	1012.395	1322.311	1673.550	2066.112
0	2025.000	1600.000	1225.000	900.000	625.000	400.000	225.000	100.000	25.000	0.000	25.000	100.000	225.000	400.000	625.000	900.000	1225.000	1600.000	2025.000
0.1	3086.414	2499.995	1975.305	1512.343	1111.109	771.603	493.826	277.777	123.457	30.864	0.000	30.864	123.457	277.777	493.826	771.603	1111.109	1512.343	1975.305
0.2	4725.958	3905.750	3163.658	2499.680	1913.818	1406.070	976.438	624.920	351.518	156.230	39.058	0.000	39.058	156.230	351.518	624.920	976.438	1406.070	1913.818
0.3	7336.231	6164.472	5094.605	4126.630	3260.547	2496.356	1834.058	1273.651	815.137	458.514	203.784	50.946	0.000	50.946	203.784	458.514	815.137	1273.651	1834.058
0.4	11640.166	9918.248	8334.083	6887.672	5579.014	4408.110	3374.959	2479.562	1721.918	1102.028	619.890	275.507	68.877	0.000	68.877	275.507	619.890	1102.028	1721.918
0.5	18992.285	16376.001	13953.516	11724.829	9689.941	7848.853	6201.562	4748.071	3488.379	2422.485	1550.391	872.095	387.598	96.899	0.000	96.899	387.598	872.095	1550.391
0.6	31952.278	27833.984	23999.711	20449.458	17183.225	14201.012	11502.820	9088.648	6958.496	5112.364	3550.253	2272.162	1278.091	568.040	142.010	0.000	142.010	568.040	1278.091
0.7	55363.078	48658.955	42387.357	36548.282	31141.731	26167.705	21626.202	17517.224	13840.770	10596.839	7785.433	5406.551	3460.192	1946.358	865.048	216.262	0.000	216.262	865.048
0.8	98337.935	87109.036	76560.676	66692.856	57505.575	48998.833	41172.630	34026.967	27561.844	21777.259	16673.214	12249.708	8506.742	5444.315	3062.427	1361.079	340.270	0.000	340.270
									$h = 2s$										
-0.8	3.369	0.000	2.038	5.817	8.672	9.271	7.562	4.448	1.381	0.000	1.880	8.402	20.700	39.667	65.980	100.139	142.506	193.334	252.797
-0.7	20.982	4.268	0.000	2.401	6.476	8.897	8.360	5.388	1.783	0.000	2.628	12.056	30.307	58.998	99.366	152.322	218.517	298.404	392.283
-0.6	59.361	21.848	4.306	0.000	2.197	5.462	6.596	4.897	1.775	0.000	2.919	13.855	35.748	71.005	121.488	188.571	273.231	376.135	497.718
-0.5	120.289	56.895	20.291	3.838	0.000	1.706	3.727	3.556	1.485	0.000	2.850	14.163	37.814	77.091	134.583	212.214	311.349	432.905	577.468
-0.4	204.943	111.676	51.184	17.517	3.131	0.000	1.127	1.946	1.045	0.000	2.530	13.395	37.443	78.991	141.535	227.715	339.397	477.828	643.781
-0.3	316.715	189.073	99.834	43.907	14.203	2.340	0.000	0.585	0.568	0.000	2.037	11.817	35.191	77.723	144.111	237.974	361.898	517.598	706.118
-0.2	462.399	294.080	170.117	86.196	35.833	10.682	1.547	0.000	0.172	0.000	1.433	9.577	31.246	73.520	142.716	243.871	380.649	555.484	769.839
-0.1	653.093	434.854	267.987	148.760	71.245	27.295	7.152	0.818	0.000	0.000	0.795	6.824	25.648	66.116	136.728	244.605	395.081	591.753	836.775
0	906.215	624.390	402.852	238.235	125.000	55.172	18.578	3.846	0.248	0.000	0.248	3.846	18.578	55.172	125.000	238.235	402.852	624.390	906.215
0.1	1249.998	883.976	590.182	365.398	204.248	98.765	38.314	10.194	1.187	0.000	0.000	1.222	10.684	40.775	106.428	222.222	400.326	649.597	975.608
0.2	1732.138	1249.840	856.460	548.710	321.110	165.420	70.304	21.549	3.225	0.000	0.387	0.000	3.480	24.035	80.623	193.941	382.764	661.680	1040.397
0.3	2435.388	1785.186	1248.178	820.766	497.035	268.065	121.371	40.757	7.027	0.000	1.959	2.018	0.000	8.071	48.987	151.436	344.325	652.109	1092.343
0.4	3505.031	2601.508	1847.828	1239.781	770.582	430.060	203.857	72.928	13.775	0.000	5.687	10.596	6.138	0.000	17.049	95.368	278.666	608.015	1115.803
0.5	5197.213	3896.149	2802.140	1909.927	1211.243	693.821	340.330	127.465	25.650	0.000	13.365	32.003	33.542	15.350	0.000	34.538	182.618	512.056	1082.600
0.6	7963.491	6018.159	4371.695	3017.133	1943.804	1136.081	571.963	221.674	46.701	0.000	28.402	78.350	105.530	87.391	35.151	0.000	68.896	349.563	949.774
0.7	12596.655	9582.071	7016.826	4891.220	3190.751	1894.494	973.179	387.121	84.395	0.000	57.246	173.010	268.463	285.704	207.944	77.083	0.000	137.037	673.739
0.8	20476.537	15660.051	11542.994	8111.293	5344.384	3213.038	1676.713	680.539	152.275	0.000	111.901	360.286	612.485	750.940	702.392	471.143	165.081	0.000	272.890
									$h = 3s$										
-0.8	2.707	0.000	0.994	2.065	2.124	1.451	0.669	0.176	0.014	0.000	0.019	0.329	1.776	5.799	14.256	29.111	52.140	84.736	127.837
-0.7	16.482	2.714	0.000	0.861	1.604	1.405	0.743	0.214	0.018	0.000	0.026	0.474	2.610	8.679	21.648	44.721	80.843	132.341	200.793
-0.6	45.068	13.647	2.100	0.000	0.548	0.869	0.589	0.195	0.018	0.000	0.029	0.546	3.093	10.519	26.715	55.990	102.369	169.075	258.303
-0.5	87.642	34.585	9.759	1.377	0.000	0.272	0.334	0.142	0.015	0.000	0.028	0.559	3.288	11.510	29.907	63.818	118.340	197.606	304.455
-0.4	142.861	65.675	24.105	6.220	0.781	0.000	0.101	0.078	0.010	0.000	0.025	0.530	3.273	11.896	31.824	69.472	131.126	221.971	345.595
-0.3	211.246	107.276	45.822	15.350	3.517	0.374	0.000	0.023	0.006	0.000	0.020	0.469	3.093	11.814	32.825	73.784	142.434	245.306	386.971
-0.2	295.760	160.945	75.917	29.563	8.777	1.699	0.139	0.000	0.002	0.000	0.014	0.381	2.762	11.285	32.968	76.979	152.974	269.326	431.989
-0.1	401.950	229.916	116.237	49.957	17.218	4.310	0.641	0.033	0.000	0.000	0.008	0.272	2.280	10.250	32.060	78.736	162.514	294.402	482.411
0	538.746	319.750	170.003	78.303	29.762	8.637	1.660	0.154	0.002	0.000	0.002	0.154	1.660	8.637	29.762	78.303	170.003	319.750	538.746
0.1	720.639	439.786	242.768	117.619	47.893	15.312	3.406	0.406	0.012	0.000	0.000	0.049	0.958	6.439	25.720	74.627	173.638	343.455	600.444
0.2	971.976	605.983	344.191	173.203	74.179	25.392	6.215	0.857	0.032	0.000	0.004	0.000	0.313	3.822	19.748	66.516	170.814	362.127	665.460
0.3	1334.655	846.055	491.251	254.478	113.213	40.746	10.669	1.617	0.070	0.000	0.020	0.081	0.000	1.289	12.130	52.943	158.038	369.991	728.582
0.4	1881.572	1208.506	713.908	378.238	173.261	64.766	17.820	2.887	0.137	0.000	0.057	0.423	0.552	0.000	4.252	33.864	131.237	357.565	777.797
0.5	2740.091	1778.452	1065.056	574.364	269.165	103.594	29.590	5.032	0.256	0.000	0.133	1.276	3.008	2.452	0.000	12.390	87.828	311.267	788.780
0.6	4132.852	2705.201	1637.910	895.839	427.443	168.308	49.484	8.731	0.465	0.000	0.283	3.117	9.428	13.897	8.766	0.000	33.596	218.346	721.090
0.7	6447.692	4249.618	2595.961	1436.047	695.128	278.704	83.814	15.213	0.841	0.000	0.571	6.865	23.865	45.117	51.491	27.651	0.000	87.152	529.241
0.8	10354.767	6863.624	4223.370	2357.975	1154.730	469.731	143.817	26.688	1.516	0.000	1.115	14.260	54.149	117.556	172.046	167.295	80.499	0.000	219.282

Table A4.: Percent IMSFE due to an SLS outlier in the SARMA(1, 1) model($n = 120, \delta = 5\sigma_a$)-b

Θ ↓ Φ →	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
									$h = s$										
-0.8	16.803	0.000	16.803	67.214	151.231	268.855	420.086	604.924	823.369	1075.420	1361.079	1680.344	2033.216	2419.695	2839.781	3293.474	3780.774	4301.681	4856.194
-0.7	107.757	26.939	0.000	26.939	107.757	242.453	431.027	673.480	969.812	1320.021	1724.110	2182.076	2693.921	3259.645	3879.247	4552.727	5280.086	6061.323	6896.439
-0.6	319.523	142.010	35.503	0.000	35.503	142.010	319.523	568.040	887.563	1278.091	1739.624	2272.162	2875.705	3550.253	4295.806	5112.364	5999.928	6958.496	7988.069
-0.5	689.063	387.598	172.266	43.066	0.000	43.066	172.266	387.598	689.063	1076.660	1550.391	2110.254	2756.250	3488.379	4306.641	5211.035	6201.563	7278.223	8441.016
-0.4	1265.083	809.653	455.430	202.413	50.603	0.000	50.603	202.413	455.430	809.653	1265.083	1821.719	2479.562	3238.612	4098.868	5060.330	6123.000	7286.876	8551.959
-0.3	2127.073	1477.134	945.366	531.768	236.341	59.085	0.000	59.085	236.341	531.768	945.366	1477.134	2127.073	2895.182	3781.463	4785.914	5908.536	7149.328	8508.291
-0.2	3402.342	2499.680	1735.889	1110.969	624.920	277.742	69.436	0.000	69.436	277.742	624.920	1110.969	1735.889	2499.680	3402.342	4443.876	5624.280	6943.556	8401.702
-0.1	5289.246	4049.579	2975.201	2066.112	1322.311	743.800	330.578	82.644	0.000	82.644	330.578	743.800	1322.311	2066.112	2975.201	4049.579	5289.246	6694.201	8264.446
0	8100.000	6400.000	4900.000	3600.000	2500.000	1600.000	900.000	400.000	100.000	0.000	100.000	400.000	900.000	1600.000	2500.000	3600.000	4900.000	6400.000	8100.000
0.1	12345.654	9999.980	7901.219	6049.371	4444.436	3086.414	1975.305	1111.109	493.826	123.457	0.000	123.457	493.826	1111.109	1975.305	3086.414	4444.436	6049.371	7901.219
0.2	18903.830	15623.000	12654.630	9998.720	7655.270	5624.280	3905.750	2499.680	1406.070	624.920	156.230	0.000	156.230	624.920	1406.070	2499.680	3905.750	5624.280	7655.270
0.3	29344.923	24657.887	20378.419	16506.519	13042.188	9985.425	7336.231	5094.605	3260.547	1834.058	815.137	203.784	0.000	203.784	815.137	1834.058	3260.547	5094.605	7336.231
0.4	46560.663	39672.991	33336.333	27550.688	22316.057	17632.440	13499.837	9918.248	6887.672	4408.110	2479.562	1102.028	275.507	0.000	275.507	1102.028	2479.562	4408.110	6887.672
0.5	75969.141	65504.004	55814.063	46899.316	38759.766	31395.410	24806.250	18992.285	13953.516	9689.941	6201.563	3488.379	1550.391	387.598	0.000	387.598	1550.391	3488.379	6201.563
0.6	127809.110	111335.936	95998.843	81797.830	68732.899	56804.049	46011.280	36354.591	27833.984	20449.458	14201.012	9088.648	5112.364	2272.162	568.040	0.000	568.040	2272.162	5112.364
0.7	221452.313	194635.822	169549.427	146193.128	124566.926	104670.820	86504.810	70068.896	55363.078	42387.357	31141.731	21626.202	13840.770	7785.433	3460.192	865.048	0.000	865.048	3460.192
0.8	393351.742	348436.145	306242.706	266771.424	230022.299	195995.332	164690.522	136107.869	110247.374	87109.036	66692.856	48998.833	34026.967	21777.259	12249.708	5444.315	1361.079	0.000	1361.079
									$h = 2s$										
-0.8	13.476	0.000	8.152	23.266	34.686	37.083	30.246	17.792	5.526	0.000	7.520	33.607	82.801	158.669	263.920	400.558	570.024	773.336	1011.187
-0.7	83.926	17.070	0.000	9.602	25.903	35.589	33.442	21.551	7.131	0.000	10.513	48.223	121.226	235.992	397.464	609.287	874.068	1193.614	1569.134
-0.6	237.444	87.391	17.224	0.000	8.788	21.848	26.383	19.588	7.101	0.000	11.675	55.419	142.991	284.020	485.951	754.283	1092.924	1504.540	1990.873
-0.5	481.156	227.580	81.164	15.350	0.000	6.822	14.908	14.224	5.940	0.000	11.400	56.651	151.258	308.365	538.330	848.856	1245.396	1731.622	2309.873
-0.4	819.774	446.705	204.734	70.066	12.526	0.000	4.509	7.785	4.178	0.000	10.121	53.580	149.772	315.962	566.142	910.859	1357.588	1911.312	2575.125
-0.3	1266.860	756.293	399.336	175.630	56.813	9.360	0.000	2.340	2.273	0.000	8.150	47.268	140.762	310.892	576.443	951.894	1447.591	2070.394	2824.474
-0.2	1849.595	1176.320	680.468	344.783	143.330	42.730	6.187	0.000	0.687	0.000	5.733	38.309	124.984	294.080	570.863	975.485	1522.595	2221.938	3079.357
-0.1	2612.371	1739.416	1071.947	595.040	284.981	109.182	28.608	3.273	0.000	0.000	3.179	27.295	102.593	264.462	546.912	978.422	1580.323	2367.010	3347.101
0	3624.862	2497.561	1611.409	952.941	500.000	220.690	74.312	15.385	0.990	0.000	0.990	15.385	74.312	220.690	500.000	952.941	1611.409	2497.561	3624.862
0.1	4999.990	3535.905	2360.730	1461.593	816.992	395.061	153.256	40.775	4.748	0.000	0.000	4.889	42.735	163.099	425.712	888.887	1601.304	2598.387	3902.431
0.2	6928.553	4999.360	3425.839	2194.841	1284.441	661.680	281.214	86.196	12.900	0.000	1.547	0.000	13.921	96.142	322.493	775.763	1531.054	2646.720	4161.590
0.3	9741.552	7140.746	4992.713	3283.065	1988.138	1072.260	485.486	163.027	28.108	0.000	7.838	8.071	0.000	32.283	195.946	605.744	1377.300	2608.438	4369.373
0.4	14020.125	10406.030	7391.314	4959.124	3082.328	1720.238	815.426	291.713	55.101	0.000	22.748	42.386	24.550	0.000	68.195	381.471	1114.665	2432.061	4463.211
0.5	20788.853	15584.596	11208.562	7639.708	4844.971	2775.285	1361.319	509.860	102.599	0.000	53.462	128.014	134.168	61.402	0.000	138.154	730.473	2048.222	4330.401
0.6	31853.963	24072.635	17486.778	12068.532	7775.215	4544.324	2287.854	886.697	186.805	0.000	113.608	313.402	422.122	349.563	140.604	0.000	275.584	1398.254	3799.096
0.7	50386.622	38328.285	28067.304	19564.880	12763.005	7577.978	3892.716	1548.484	337.580	0.000	228.983	692.038	1073.853	1142.816	831.777	308.334	0.000	548.149	2694.958
0.8	81906.147	62640.206	46171.977	32445.173	21377.537	12852.153	6706.854	2722.157	609.102	0.000	447.603	1441.142	2449.942	3003.760	2809.566	1884.570	660.325	0.000	1091.558
									$h = 3s$										
-0.8	10.829	0.000	3.975	8.261	8.496	5.805	2.674	0.704	0.055	0.000	0.075	1.318	7.102	23.197	57.024	116.443	208.561	338.944	511.347
-0.7	65.926	10.856	0.000	3.444	6.414	5.620	2.973	0.855	0.071	0.000	0.105	1.895	10.441	34.717	86.590	178.885	323.372	529.364	803.173
-0.6	180.273	54.586	8.399	0.000	2.192	3.474	2.357	0.779	0.071	0.000	0.116	2.183	12.371	42.077	106.861	223.960	409.477	676.300	1033.213
-0.5	350.569	138.341	39.035	5.507	0.000	1.090	1.337	0.567	0.059	0.000	0.114	2.237	13.151	46.042	119.629	255.273	473.358	790.423	1217.818
-0.4	571.443	262.701	96.419	24.879	3.124	0.000	0.405	0.311	0.042	0.000	0.101	2.121	13.092	47.583	127.294	277.889	524.504	887.882	1382.379
-0.3	844.982	429.102	183.287	61.402	14.068	1.495	0.000	0.094	0.023	0.000	0.081	1.876	12.374	47.256	131.301	295.134	569.735	981.224	1547.884
-0.2	1183.039	643.781	303.670	118.250	35.108	6.795	0.556	0.000	0.007	0.000	0.057	1.524	11.050	45.141	131.874	307.916	611.894	1077.303	1727.957
-0.1	1607.799	919.665	464.948	199.827	68.870	17.241	2.566	0.131	0.000	0.000	0.032	1.088	9.120	41.002	128.241	314.946	650.055	1177.609	1929.645
0	2154.986	1279.001	680.013	313.212	119.048	34.548	6.639	0.614	0.010	0.000	0.010	0.614	6.639	34.548	119.048	313.212	680.013	1279.001	2154.986
0.1	2882.557	1759.144	971.070	470.474	191.570	61.250	13.624	1.626	0.047	0.000	0.000	0.195	3.833	25.756	102.880	298.507	694.552	1373.820	2401.774
0.2	3887.904	2423.932	1376.762	692.811	296.716	101.567	24.862	3.429	0.129	0.000	0.015	0.000	1.252	15.289	78.993	266.063	683.257	1448.507	2661.838
0.3	5338.621	3384.221	1965.003	1017.913	452.854	162.986	42.677	6.469	0.281	0.000	0.078	0.323	0.000	5.157	48.520	211.773	632.152	1479.965	2914.326
0.4	7526.288	4834.025	2855.633	1512.953	693.045	259.062	71.279	11.546	0.550	0.000	0.227	1.693	2.208	0.000	17.007	135.454	524.947	1430.261	3111.189
0.5	10960.364	7113.808	4260.223	2297.457	1076.660	414.375	118.362	20.130	1.023	0.000	0.534	5.104	12.034	9.809	0.000	49.559	351.311	1245.068	3155.122
0.6	16531.406	10820.805	6551.639	3583.355	1709.774	673.233	197.935	34.923	1.862	0.000	1.134	12.467	37.711	55.588	35.064	0.000	134.384	873.384	2884.360
0.7	25790.767	16998.472	10383.843	5744.188	2780.512	1114.817	335.258	60.850	3.363	0.000	2.284	27.462	95.462	180.466	205.964	110.606	0.000	348.607	2116.964
0.8	41419.069	27454.495	16893.480	9431.901	4618.922	1878.925	575.270	106.751	6.064	0.000	4.461	57.042	216.596	470.224	688.186	669.180	321.997	0.000	877.128

Table A5.: Percent IMSFE due to an SLS outlier in the SARMA(1, 1) model($n = 120, \delta = 10\sigma_a$)-c

$\Theta\downarrow \Phi \rightarrow$	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$h = s$																			
-0.8	1.924	0.000	1.924	7.696	17.315	30.782	48.097	69.260	94.271	123.129	155.836	192.390	232.792	277.041	325.139	377.084	432.877	492.518	556.006
-0.7	11.062	2.765	0.000	2.765	11.062	24.889	44.248	69.137	99.557	135.509	176.991	224.004	276.548	334.623	398.230	467.367	542.035	622.234	707.964
-0.6	30.587	13.594	3.399	0.000	3.399	13.594	30.587	54.376	84.963	122.347	166.527	217.505	275.280	339.852	411.221	489.387	574.350	666.110	764.667
-0.5	63.501	35.719	15.875	3.969	0.000	3.969	15.875	35.719	63.501	99.220	142.877	194.472	254.004	321.474	396.881	480.226	571.509	670.729	777.887
-0.4	114.646	73.373	41.272	18.343	4.586	0.000	4.586	18.343	41.272	73.373	114.646	165.090	224.705	293.492	371.451	458.582	554.884	660.358	775.004
-0.3	191.691	133.119	85.196	47.923	21.299	5.325	0.000	5.325	21.299	47.923	85.196	133.119	191.691	260.913	340.784	431.304	532.475	644.294	766.763
-0.2	306.248	224.999	156.249	99.999	56.250	25.000	6.250	0.000	6.250	25.000	56.250	99.999	156.249	224.999	306.248	399.998	506.247	624.997	756.246
-0.1	476.033	364.463	267.769	185.950	119.008	66.942	29.752	7.438	0.000	7.438	29.752	66.942	119.008	185.950	267.769	364.463	476.033	602.479	743.802
0	729.000	576.000	441.000	324.000	225.000	144.000	81.000	36.000	9.000	0.000	9.000	36.000	81.000	144.000	225.000	324.000	441.000	576.000	729.000
0.1	1111.111	900.000	711.111	544.444	400.000	277.778	177.778	100.000	44.444	11.111	0.000	11.111	44.444	100.000	177.778	277.778	400.000	544.444	711.111
0.2	1701.554	1406.243	1139.057	899.995	689.059	506.247	351.561	224.999	126.562	56.250	14.062	0.000	14.062	56.250	126.562	224.999	351.561	506.247	689.059
0.3	2644.551	2222.157	1836.494	1487.560	1175.356	899.882	661.138	459.123	293.839	165.284	73.460	18.365	0.000	18.365	73.460	165.284	293.839	459.123	661.138
0.4	4219.464	3595.283	3021.036	2496.724	2022.347	1597.904	1223.395	898.821	624.181	399.476	224.705	99.869	24.967	0.000	24.967	99.869	224.705	399.476	624.181
0.5	7000.983	6036.562	5143.579	4322.035	3571.930	2893.263	2286.035	1750.246	1285.895	892.982	571.509	321.474	142.877	35.719	0.000	35.719	142.877	321.474	571.509
0.6	12234.668	10657.755	9189.595	7830.187	6579.532	5437.630	4404.480	3480.083	2664.439	1957.547	1359.408	870.021	489.387	217.505	54.376	0.000	54.376	217.505	489.387
0.7	22733.498	19980.614	17405.335	15007.661	12787.593	10745.130	8880.273	7193.021	5683.375	4351.334	3196.898	2220.068	1420.844	799.225	355.211	88.803	0.000	88.803	355.211
0.8	45036.522	39893.943	35063.036	30543.800	26336.236	22440.343	18856.122	15583.571	12622.693	9973.486	7635.950	5610.086	3895.893	2493.371	1402.521	623.343	155.836	0.000	155.836
$h = 2s$																			
-0.8	1.543	0.000	0.933	2.664	3.971	4.246	3.463	2.037	0.633	0.000	0.861	3.848	9.480	18.167	30.217	45.862	65.265	88.543	115.775
-0.7	8.616	1.752	0.000	0.986	2.659	3.653	3.433	2.212	0.732	0.000	1.079	4.950	12.445	24.226	40.802	62.547	89.729	122.532	161.082
-0.6	22.730	8.366	1.649	0.000	0.841	2.091	2.526	1.875	0.680	0.000	1.118	5.305	13.688	27.188	46.518	72.205	104.621	144.024	190.578
-0.5	44.341	20.973	7.480	1.415	0.000	0.629	1.374	1.311	0.547	0.000	1.051	5.221	13.939	28.418	49.610	78.227	114.770	159.579	212.868
-0.4	74.290	40.482	18.554	6.350	1.135	0.000	0.409	0.706	0.379	0.000	0.917	4.856	13.573	28.633	51.305	82.545	123.029	173.209	233.365
-0.3	114.169	68.157	35.988	15.828	5.120	0.844	0.000	0.211	0.205	0.000	0.734	4.260	12.685	28.017	51.949	85.784	130.456	186.583	254.540
-0.2	166.484	105.882	61.250	31.034	12.901	3.846	0.557	0.000	0.062	0.000	0.516	3.448	11.250	26.470	51.384	87.804	137.050	199.999	277.176
-0.1	235.114	156.548	96.475	53.554	25.648	9.826	2.575	0.295	0.000	0.000	0.286	2.457	9.233	23.802	49.222	88.058	142.229	213.031	301.240
0	326.238	224.780	145.027	85.765	45.000	19.862	6.688	1.385	0.089	0.000	0.089	1.385	6.688	19.862	45.000	85.765	145.027	224.780	326.238
0.1	450.000	318.232	212.466	131.544	73.529	35.556	13.793	3.670	0.427	0.000	0.000	0.440	3.846	14.679	38.314	80.000	144.118	233.855	351.220
0.2	623.646	449.998	308.363	197.560	115.614	59.559	25.312	7.759	1.161	0.000	0.139	0.000	1.253	8.654	29.028	69.827	137.812	238.234	374.589
0.3	877.904	643.521	449.941	295.868	179.170	96.632	43.752	14.692	2.533	0.000	0.706	0.727	0.000	2.909	17.659	54.589	124.122	235.071	393.766
0.4	1270.545	943.025	669.823	449.410	279.330	155.893	73.896	26.436	4.993	0.000	2.062	3.841	2.225	0.000	6.180	34.570	101.014	220.400	404.469
0.5	1915.809	1436.208	1032.932	704.042	446.491	255.758	125.453	46.986	9.455	0.000	4.927	11.797	12.364	5.659	0.000	12.732	67.317	188.755	399.071
0.6	3049.256	2304.379	1673.941	1155.274	744.291	435.010	219.007	84.880	17.882	0.000	10.875	30.001	40.408	33.462	13.459	0.000	26.381	133.849	363.673
0.7	5172.509	3934.644	2881.288	2008.460	1310.204	777.928	399.612	158.962	34.655	0.000	23.507	71.042	110.238	117.317	85.387	31.652	0.000	56.271	276.655
0.8	9377.785	7171.945	5286.427	3714.787	2447.606	1471.498	767.896	311.671	69.739	0.000	51.248	165.003	280.504	343.913	321.679	215.773	75.603	0.000	124.977
$h = 3s$																			
-0.8	1.240	0.000	0.455	0.946	0.973	0.665	0.306	0.081	0.006	0.000	0.009	0.151	0.813	2.656	6.529	13.332	23.879	38.807	58.546
-0.7	6.768	1.114	0.000	0.354	0.658	0.577	0.305	0.088	0.007	0.000	0.011	0.195	1.072	3.564	8.889	18.364	33.196	54.343	82.451
-0.6	17.257	5.225	0.804	0.000	0.210	0.333	0.226	0.075	0.007	0.000	0.011	0.209	1.184	4.028	10.229	21.439	39.198	64.740	98.905
-0.5	32.307	12.749	3.597	0.507	0.000	0.100	0.123	0.052	0.005	0.000	0.010	0.206	1.212	4.243	11.024	23.525	43.623	72.842	112.229
-0.4	51.786	23.807	8.738	2.255	0.283	0.000	0.037	0.028	0.004	0.000	0.009	0.192	1.186	4.312	11.536	25.183	47.532	80.462	125.275
-0.3	76.149	38.670	16.518	5.533	1.268	0.135	0.000	0.008	0.002	0.000	0.007	0.169	1.115	4.259	11.833	26.597	51.344	88.427	139.495
-0.2	106.487	57.947	27.334	10.644	3.160	0.612	0.050	0.000	0.001	0.000	0.005	0.137	0.995	4.063	11.870	27.716	55.077	96.969	155.535
-0.1	144.702	82.770	41.845	17.984	6.198	1.552	0.231	0.012	0.000	0.000	0.003	0.098	0.821	3.690	11.542	28.345	58.505	105.985	173.668
0	193.949	115.110	61.201	28.189	10.714	3.109	0.597	0.055	0.001	0.000	0.001	0.055	0.597	3.109	10.714	28.189	61.201	115.110	193.949
0.1	259.431	158.323	87.396	42.343	17.241	5.512	1.226	0.146	0.004	0.000	0.000	0.018	0.345	2.318	9.259	26.866	62.510	123.644	216.160
0.2	349.954	218.181	123.924	62.361	26.708	9.142	2.238	0.309	0.012	0.000	0.001	0.000	0.113	1.376	7.110	23.949	61.501	130.382	239.595
0.3	481.114	304.984	177.085	91.734	40.811	14.688	3.846	0.583	0.025	0.000	0.007	0.029	0.000	0.465	4.373	19.085	56.969	133.374	262.638
0.4	682.054	438.074	258.786	137.108	62.806	23.477	6.459	1.046	0.050	0.000	0.021	0.153	0.200	0.000	1.541	12.275	47.572	129.614	281.945
0.5	1010.059	655.577	392.603	211.724	99.220	38.187	10.908	1.855	0.094	0.000	0.049	0.470	1.109	0.904	0.000	4.567	32.375	114.740	290.762
0.6	1582.487	1035.833	627.163	343.021	163.670	64.446	18.948	3.343	0.178	0.000	0.109	1.193	3.610	5.321	3.357	0.000	12.864	83.606	276.109
0.7	2647.587	1745.002	1065.968	589.678	285.437	114.443	34.416	6.247	0.345	0.000	0.234	2.819	9.800	18.526	21.144	11.354	0.000	35.787	217.320
0.8	4742.246	3143.382	1934.207	1079.899	528.840	215.126	65.865	12.222	0.694	0.000	0.511	6.531	24.799	53.838	78.793	76.617	36.867	0.000	100.426

Table A6.: Percent IMSFE due to an SLS outlier in the SARMA(1, 1) model($n = 180, \delta = 3\sigma_a$)-d

Θ ↓ Φ →	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$h = s$																			
-0.8	5.344	0.000	5.344	21.377	48.097	85.507	133.604	192.390	261.864	342.026	432.877	534.416	646.643	769.559	903.163	1047.455	1202.436	1368.105	1544.462
-0.7	30.728	7.682	0.000	7.682	30.728	69.137	122.910	192.047	276.548	376.413	491.641	622.234	768.190	929.510	1106.193	1298.241	1505.652	1728.427	1966.566
-0.6	84.963	37.761	9.440	0.000	9.440	37.761	84.963	151.045	236.008	339.852	462.576	604.181	764.667	944.033	1142.280	1359.408	1595.416	1850.305	2124.074
-0.5	176.392	99.220	44.098	11.024	0.000	11.024	44.098	99.220	176.392	275.612	396.881	540.199	705.566	892.982	1102.448	1333.961	1587.524	1863.136	2160.797
-0.4	318.460	203.814	114.646	50.954	12.738	0.000	12.738	50.954	114.646	203.814	318.460	458.582	624.181	815.257	1031.810	1273.839	1541.345	1834.328	2152.788
-0.3	532.475	369.774	236.655	133.119	59.164	14.791	0.000	14.791	59.164	133.119	236.655	369.774	532.475	724.757	946.621	1198.068	1479.096	1789.706	2129.898
-0.2	850.690	624.997	434.026	277.776	156.249	69.444	17.361	0.000	17.361	69.444	156.249	277.776	434.026	624.997	850.690	1111.105	1406.243	1736.102	2100.684
-0.1	1322.314	1012.397	743.802	516.529	330.579	185.950	82.645	20.661	0.000	20.661	82.645	185.950	330.579	516.529	743.802	1012.397	1322.314	1673.554	2066.116
0	2025.000	1600.000	1225.000	900.000	625.000	400.000	225.000	100.000	25.000	0.000	25.000	100.000	225.000	400.000	625.000	900.000	1225.000	1600.000	2025.000
0.1	3086.420	2500.000	1975.309	1512.346	1111.111	771.605	493.827	277.778	123.457	30.864	0.000	30.864	123.457	277.778	493.827	771.605	1111.111	1512.346	1975.309
0.2	4726.538	3906.230	3164.046	2499.987	1914.053	1406.243	976.558	624.997	351.561	156.249	39.062	0.000	39.062	156.249	351.561	624.997	976.558	1406.243	1914.053
0.3	7345.975	6172.659	5101.371	4132.111	3264.878	2499.672	1836.494	1275.343	816.219	459.123	204.055	51.014	0.000	51.014	204.055	459.123	816.219	1275.343	1836.494
0.4	11720.733	9986.897	8391.768	6935.345	5617.630	4438.621	3398.319	2496.724	1733.836	1109.655	624.181	277.414	69.353	0.000	69.353	277.414	624.181	1109.655	1733.836
0.5	19447.174	16768.227	14287.720	12005.653	9922.028	8036.842	6350.098	4861.794	3571.930	2480.507	1587.524	892.982	396.881	99.220	0.000	99.220	396.881	892.982	1587.524
0.6	33985.188	29604.875	25526.652	21750.520	18276.479	15104.528	12234.668	9666.898	7401.219	5437.630	3776.132	2416.724	1359.408	604.181	151.045	0.000	151.045	604.181	1359.408
0.7	63148.606	55501.705	48348.152	41687.947	35521.091	29847.583	24667.424	19980.614	15787.152	12087.038	8880.273	6166.856	3946.788	2220.068	986.697	246.674	0.000	246.674	986.697
0.8	125101.449	110816.508	97397.322	84843.889	73156.211	62334.286	52378.115	43287.699	35063.036	27704.127	21210.972	15583.571	10821.925	6926.032	3895.893	1731.508	432.877	0.000	432.877
$h = 2s$																			
-0.8	4.286	0.000	2.593	7.400	11.032	11.794	9.619	5.659	1.757	0.000	2.392	10.688	26.334	50.463	83.937	127.393	181.290	245.951	321.598
-0.7	23.932	4.868	0.000	2.738	7.386	10.149	9.536	6.146	2.033	0.000	2.998	13.751	34.569	67.295	113.339	173.742	249.246	340.367	447.449
-0.6	63.138	23.238	4.580	0.000	2.337	5.809	7.015	5.208	1.888	0.000	3.105	14.736	38.022	75.523	129.217	200.568	290.615	400.066	529.385
-0.5	123.170	58.258	20.777	3.930	0.000	1.746	3.816	3.641	1.521	0.000	2.918	14.502	38.720	78.938	137.806	217.297	318.806	443.274	591.299
-0.4	206.362	112.449	51.538	17.638	3.153	0.000	1.135	1.960	1.052	0.000	2.548	13.488	37.702	79.537	142.515	229.291	341.746	481.135	648.237
-0.3	317.136	189.324	99.966	43.966	14.222	2.343	0.000	0.586	0.569	0.000	2.040	11.833	35.237	77.826	144.302	238.290	362.378	518.286	707.056
-0.2	462.456	294.116	170.138	86.206	35.837	10.684	1.547	0.000	0.172	0.000	1.433	9.578	31.250	73.529	142.733	243.901	380.696	555.553	769.934
-0.1	653.094	434.855	267.987	148.760	71.245	27.295	7.152	0.818	0.000	0.000	0.795	6.824	25.648	66.116	136.728	244.606	395.082	591.754	836.777
0	906.215	624.390	402.852	238.235	125.000	55.172	18.578	3.846	0.248	0.000	0.248	3.846	18.578	55.172	125.000	238.235	402.852	624.390	906.215
0.1	1250.000	883.978	590.184	365.399	204.248	98.765	38.314	10.194	1.187	0.000	0.000	1.222	10.684	40.775	106.428	222.222	400.327	649.598	975.610
0.2	1732.351	1249.994	856.565	548.778	321.150	165.440	70.312	21.552	3.225	0.000	0.387	0.000	3.481	24.038	80.633	193.965	382.811	661.761	1040.525
0.3	2438.623	1787.557	1249.836	821.856	497.695	268.421	121.533	40.811	7.036	0.000	1.962	2.020	0.000	8.081	49.052	151.637	344.782	652.976	1093.794
0.4	3529.291	2619.514	1860.618	1248.362	775.916	433.036	205.268	73.433	13.871	0.000	5.726	10.670	6.180	0.000	17.167	96.028	280.595	612.224	1123.526
0.5	5321.693	3989.467	2869.255	1955.672	1240.253	710.439	348.481	130.518	26.264	0.000	13.686	32.770	34.345	15.718	0.000	35.366	186.992	524.320	1108.530
0.6	8470.155	6401.054	4649.836	3209.093	2067.475	1208.362	608.354	235.778	49.673	0.000	30.209	83.335	112.245	92.951	37.387	0.000	73.279	371.804	1010.202
0.7	14368.082	10929.566	8003.579	5579.056	3639.456	2160.911	1110.034	441.561	96.263	0.000	65.296	197.339	306.216	325.882	237.187	87.923	0.000	156.308	768.485
0.8	26049.402	19922.069	14684.519	10318.851	6798.904	4087.494	2133.045	865.754	193.718	0.000	142.356	458.340	779.179	955.315	893.553	599.368	210.010	0.000	347.159
$h = 3s$																			
-0.8	3.444	0.000	1.264	2.627	2.702	1.846	0.850	0.224	0.018	0.000	0.024	0.419	2.259	7.377	18.136	37.034	66.331	107.798	162.628
-0.7	18.799	3.096	0.000	0.982	1.829	1.603	0.848	0.244	0.020	0.000	0.030	0.540	2.977	9.900	24.692	51.010	92.212	150.952	229.030
-0.6	47.936	14.515	2.233	0.000	0.583	0.924	0.627	0.207	0.019	0.000	0.031	0.580	3.290	11.189	28.415	59.552	108.882	179.832	274.737
-0.5	89.741	35.414	9.992	1.410	0.000	0.279	0.342	0.145	0.015	0.000	0.029	0.573	3.367	11.786	30.624	65.347	121.174	202.339	311.747
-0.4	143.850	66.130	24.272	6.263	0.786	0.000	0.102	0.078	0.011	0.000	0.025	0.534	3.296	11.978	32.044	69.953	132.034	223.507	347.987
-0.3	211.526	107.418	45.883	15.371	3.522	0.374	0.000	0.023	0.006	0.000	0.020	0.470	3.098	11.830	32.869	73.882	142.623	245.632	387.485
-0.2	295.796	160.965	75.927	29.566	8.778	1.699	0.139	0.000	0.002	0.000	0.014	0.381	2.763	11.287	32.972	76.988	152.992	269.359	432.042
-0.1	401.951	229.917	116.237	49.957	17.218	4.310	0.641	0.033	0.000	0.000	0.008	0.272	2.280	10.250	32.060	78.737	162.514	294.403	482.412
0	538.746	319.750	170.003	78.303	29.762	8.637	1.660	0.154	0.002	0.000	0.002	0.154	1.660	8.637	29.762	78.303	170.003	319.750	538.746
0.1	720.641	439.787	242.768	117.619	47.893	15.312	3.406	0.406	0.012	0.000	0.000	0.049	0.958	6.439	25.720	74.627	173.638	343.456	600.445
0.2	972.095	606.058	344.233	173.224	74.188	25.395	6.216	0.857	0.032	0.000	0.004	0.000	0.313	3.823	19.751	66.524	170.835	362.171	665.541
0.3	1336.428	847.179	491.903	254.816	113.364	40.801	10.683	1.619	0.070	0.000	0.020	0.081	0.000	1.291	12.146	53.014	158.248	370.483	729.549
0.4	1894.595	1216.871	718.850	380.856	174.461	65.214	17.943	2.907	0.138	0.000	0.057	0.426	0.556	0.000	4.281	34.098	132.145	360.040	783.181
0.5	2805.720	1821.048	1090.565	588.121	275.612	106.075	30.299	5.153	0.262	0.000	0.137	1.306	3.080	2.511	0.000	12.686	89.931	318.722	807.673
0.6	4395.797	2877.315	1742.119	952.835	454.639	179.017	52.632	9.286	0.495	0.000	0.301	3.315	10.028	14.781	9.324	0.000	35.734	232.238	766.968
0.7	7354.409	4847.228	2961.022	1637.994	792.881	317.897	95.601	17.352	0.959	0.000	0.651	7.831	27.222	51.461	58.732	31.540	0.000	99.407	603.666
0.8	13172.906	8731.618	5372.797	2999.718	1469.000	597.573	182.959	33.951	1.929	0.000	1.419	18.142	68.886	149.550	218.870	212.826	102.408	0.000	278.961

Table A7.: Percent IMSFE due to an SLS outlier in the SARMA(1, 1) model($n = 180, \delta = 5\sigma_a$)-e

Θ ↓ Φ →	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
									$h = s$										
-0.8	21.377	0.000	21.377	85.507	192.390	342.026	534.416	769.559	1047.455	1368.105	1731.508	2137.664	2586.574	3078.236	3612.652	4189.822	4809.744	5472.420	6177.849
-0.7	122.910	30.728	0.000	30.728	122.910	276.548	491.641	768.190	1106.193	1505.652	1966.566	2488.935	3072.759	3718.038	4424.773	5192.962	6022.607	6913.707	7866.262
-0.6	339.852	151.045	37.761	0.000	37.761	151.045	339.852	604.181	944.033	1359.408	1850.305	2416.724	3058.667	3776.132	4569.120	5437.630	6381.663	7401.219	8496.297
-0.5	705.566	396.881	176.392	44.098	0.000	44.098	176.392	396.881	705.566	1102.448	1587.524	2160.797	2822.266	3571.930	4409.790	5335.846	6350.098	7452.545	8643.188
-0.4	1273.839	815.257	458.582	203.814	50.954	0.000	50.954	203.814	458.582	815.257	1273.839	1834.328	2496.724	3261.028	4127.238	5095.356	6165.380	7337.312	8611.151
-0.3	2129.898	1479.096	946.621	532.475	236.655	59.164	0.000	59.164	236.655	532.475	946.621	1479.096	2129.898	2899.028	3786.485	4792.271	5916.383	7158.824	8519.592
-0.2	3402.760	2499.987	1736.102	1111.105	624.997	277.776	69.444	0.000	69.444	277.776	624.997	1111.105	1736.102	2499.987	3402.760	4444.422	5624.971	6944.409	8402.735
-0.1	5289.256	4049.587	2975.207	2066.116	1322.314	743.802	330.579	82.645	0.000	82.645	330.579	743.802	1322.314	2066.116	2975.207	4049.587	5289.256	6694.215	8264.463
0	8100.000	6400.000	4900.000	3600.000	2500.000	1600.000	900.000	400.000	100.000	0.000	100.000	400.000	900.000	1600.000	2500.000	3600.000	4900.000	6400.000	8100.000
0.1	12345.679	10000.000	7901.234	6049.383	4444.444	3086.420	1975.309	1111.111	493.827	123.457	0.000	123.457	493.827	1111.111	1975.309	3086.420	4444.444	6049.383	7901.234
0.2	18906.153	15624.920	12656.185	9999.949	7656.211	5624.971	3906.230	2499.987	1406.243	624.997	156.249	0.000	156.249	624.997	1406.243	2499.987	3906.230	5624.971	7656.211
0.3	29383.899	24690.637	20405.485	16528.443	13059.511	9998.688	7345.975	5101.371	3264.878	1836.494	816.219	204.055	0.000	204.055	816.219	1836.494	3264.878	5101.371	7345.975
0.4	46882.934	39947.588	33567.071	27741.381	22470.518	17754.484	13593.277	9986.897	6935.345	4438.621	2496.724	1109.655	277.414	0.000	277.414	1109.655	2496.724	4438.621	6935.345
0.5	77788.696	67072.906	57150.879	48022.614	39688.110	32147.369	25400.391	19447.174	14287.720	9922.028	6350.098	3571.930	1587.524	396.881	0.000	396.881	1587.524	3571.930	6350.098
0.6	135940.752	118419.499	102106.609	87002.081	73105.915	60418.112	48938.671	38667.592	29604.875	21750.520	15104.528	9666.898	5437.630	2416.724	604.181	0.000	604.181	2416.724	5437.630
0.7	252594.424	222006.818	193392.606	166751.788	142084.364	119390.333	98669.697	79922.455	63148.606	48348.152	35521.091	24667.424	15787.152	8880.273	3946.788	986.697	0.000	986.697	3946.788
0.8	500405.796	443266.034	389589.287	339375.557	292624.843	249337.144	209512.461	173150.794	140252.143	110816.508	84843.889	62334.286	43287.699	27704.127	15583.571	6926.032	1731.508	0.000	1731.508
									$h = 2s$										
-0.8	17.144	0.000	10.371	29.598	44.126	47.176	38.478	22.634	7.030	0.000	9.566	42.753	105.336	201.852	335.748	509.573	725.161	983.806	1286.390
-0.7	95.728	19.471	0.000	10.952	29.546	40.594	38.145	24.582	8.134	0.000	11.991	55.004	138.274	269.179	453.358	694.969	996.986	1361.468	1789.796
-0.6	252.550	92.951	18.320	0.000	9.347	23.238	28.061	20.834	7.552	0.000	12.418	58.944	152.088	302.091	516.869	802.273	1162.459	1600.264	2117.539
-0.5	492.680	233.031	83.108	15.718	0.000	6.986	15.265	14.564	6.082	0.000	11.673	58.008	154.880	315.751	551.224	869.188	1275.225	1773.096	2365.197
-0.4	825.448	449.797	206.152	70.551	12.612	0.000	4.540	7.839	4.207	0.000	10.191	53.951	150.809	318.149	570.061	917.164	1366.985	1924.541	2592.949
-0.3	1268.542	757.297	399.866	175.863	56.888	9.372	0.000	2.343	2.276	0.000	8.161	47.331	140.949	311.305	577.208	953.159	1449.514	2073.144	2828.225
-0.2	1849.823	1176.465	680.552	344.826	143.348	42.735	6.188	0.000	0.688	0.000	5.734	38.314	124.999	294.116	570.933	975.605	1522.782	2222.211	3079.735
-0.1	2612.376	1739.420	1071.949	595.041	284.981	109.182	28.608	3.273	0.000	0.000	3.179	27.295	102.593	264.463	546.913	978.424	1580.327	2367.015	3347.107
0	3624.862	2497.561	1611.409	952.941	500.000	220.690	74.312	15.385	0.990	0.000	0.990	15.385	74.312	220.690	500.000	952.941	1611.409	2497.561	3624.862
0.1	5000.000	3535.912	2360.735	1461.596	816.993	395.062	153.257	40.775	4.748	0.000	0.000	4.889	42.735	163.099	425.713	888.889	1601.307	2598.393	3902.439
0.2	6929.405	4999.974	3426.260	2195.111	1284.599	661.761	281.249	86.206	12.901	0.000	1.547	0.000	13.923	96.153	322.533	775.858	1531.242	2647.045	4162.101
0.3	9754.491	7150.230	4999.344	3287.425	1990.779	1073.685	486.131	163.244	28.145	0.000	7.848	8.081	0.000	32.326	196.207	606.548	1379.129	2611.902	4375.176
0.4	14117.166	10478.056	7442.473	4993.449	3103.663	1732.145	821.070	293.732	55.483	0.000	22.906	42.679	24.720	0.000	68.667	384.111	1122.381	2448.894	4494.104
0.5	21286.772	15957.866	11477.021	7822.688	4961.014	2841.756	1393.924	522.072	105.057	0.000	54.742	131.080	137.382	62.872	0.000	141.463	747.968	2097.280	4434.120
0.6	33880.618	25604.216	18599.345	12836.373	8269.900	4833.449	2433.415	943.112	198.690	0.000	120.836	333.341	448.979	371.804	149.550	0.000	293.118	1487.215	4040.808
0.7	57472.327	43718.266	32014.317	22316.224	14557.824	8643.644	4440.136	1766.242	385.052	0.000	261.184	789.358	1224.865	1303.526	948.747	351.694	0.000	625.234	3073.941
0.8	104197.608	79688.276	58738.077	41275.406	27195.617	16349.977	8532.182	3463.016	774.874	0.000	569.422	1833.361	3116.714	3821.259	3574.214	2397.473	840.039	0.000	1388.635
									$h = 3s$										
-0.8	13.776	0.000	5.057	10.510	10.808	7.385	3.402	0.896	0.070	0.000	0.095	1.677	9.035	29.510	72.543	148.134	265.323	431.191	650.514
-0.7	75.197	12.383	0.000	3.929	7.316	6.410	3.391	0.975	0.081	0.000	0.119	2.161	11.909	39.600	98.767	204.041	368.847	603.807	916.120
-0.6	191.742	58.059	8.933	0.000	2.331	3.695	2.507	0.829	0.075	0.000	0.124	2.322	13.158	44.754	113.660	238.209	435.530	719.329	1098.949
-0.5	358.966	141.654	39.969	5.638	0.000	1.116	1.369	0.581	0.061	0.000	0.116	2.290	13.466	47.144	122.494	261.387	484.696	809.355	1246.987
-0.4	575.398	264.519	97.086	25.052	3.145	0.000	0.408	0.313	0.042	0.000	0.102	2.135	13.183	47.912	128.175	279.813	528.134	894.028	1391.948
-0.3	846.104	429.672	183.530	61.483	14.087	1.497	0.000	0.094	0.023	0.000	0.081	1.878	12.390	47.319	131.475	295.526	570.491	982.527	1549.940
-0.2	1183.185	643.860	303.707	118.265	35.112	6.796	0.556	0.000	0.007	0.000	0.057	1.524	11.051	45.146	131.890	307.954	611.970	1077.436	1728.170
-0.1	1607.803	919.667	464.949	199.827	68.871	17.241	2.566	0.131	0.000	0.000	0.032	1.088	9.120	41.002	128.242	314.946	650.057	1177.611	1929.649
0	2154.986	1279.001	680.013	313.212	119.048	34.548	6.639	0.614	0.010	0.000	0.010	0.614	6.639	34.548	119.048	313.212	680.013	1279.001	2154.986
0.1	2882.562	1759.148	971.072	470.475	191.571	61.250	13.624	1.626	0.047	0.000	0.000	0.195	3.833	25.756	102.881	298.507	694.553	1373.823	2401.779
0.2	3888.382	2424.230	1376.931	692.896	296.752	101.580	24.865	3.429	0.129	0.000	0.015	0.000	1.252	15.290	79.002	266.096	683.341	1448.685	2662.165
0.3	5345.712	3388.716	1967.613	1019.264	453.455	163.202	42.734	6.478	0.281	0.000	0.078	0.323	0.000	5.164	48.584	212.054	632.992	1481.930	2918.197
0.4	7578.382	4867.483	2875.398	1523.425	697.842	260.856	71.772	11.626	0.554	0.000	0.229	1.705	2.223	0.000	17.124	136.392	528.581	1440.161	3132.723
0.5	11222.879	7284.193	4362.260	2352.484	1102.448	424.300	121.196	20.612	1.048	0.000	0.547	5.226	12.322	10.044	0.000	50.746	359.725	1274.889	3230.691
0.6	17583.189	11509.260	6968.476	3811.340	1818.555	716.067	210.528	37.145	1.980	0.000	1.206	13.260	40.110	59.125	37.295	0.000	142.934	928.951	3067.873
0.7	29417.637	19388.911	11844.089	6551.974	3171.526	1271.590	382.404	69.407	3.836	0.000	2.605	31.324	108.886	205.845	234.928	126.160	0.000	397.630	2414.666
0.8	52691.624	34926.471	21491.186	11998.874	5876.001	2390.290	731.834	135.805	7.714	0.000	5.676	72.566	275.544	598.200	875.482	851.303	409.632	0.000	1115.846

Table A8.: Percent IMSFE due to an SLS outlier in the SARMA(1, 1) model($n = 180, \delta = 10\sigma_a$)-f

Appendix B. Figures

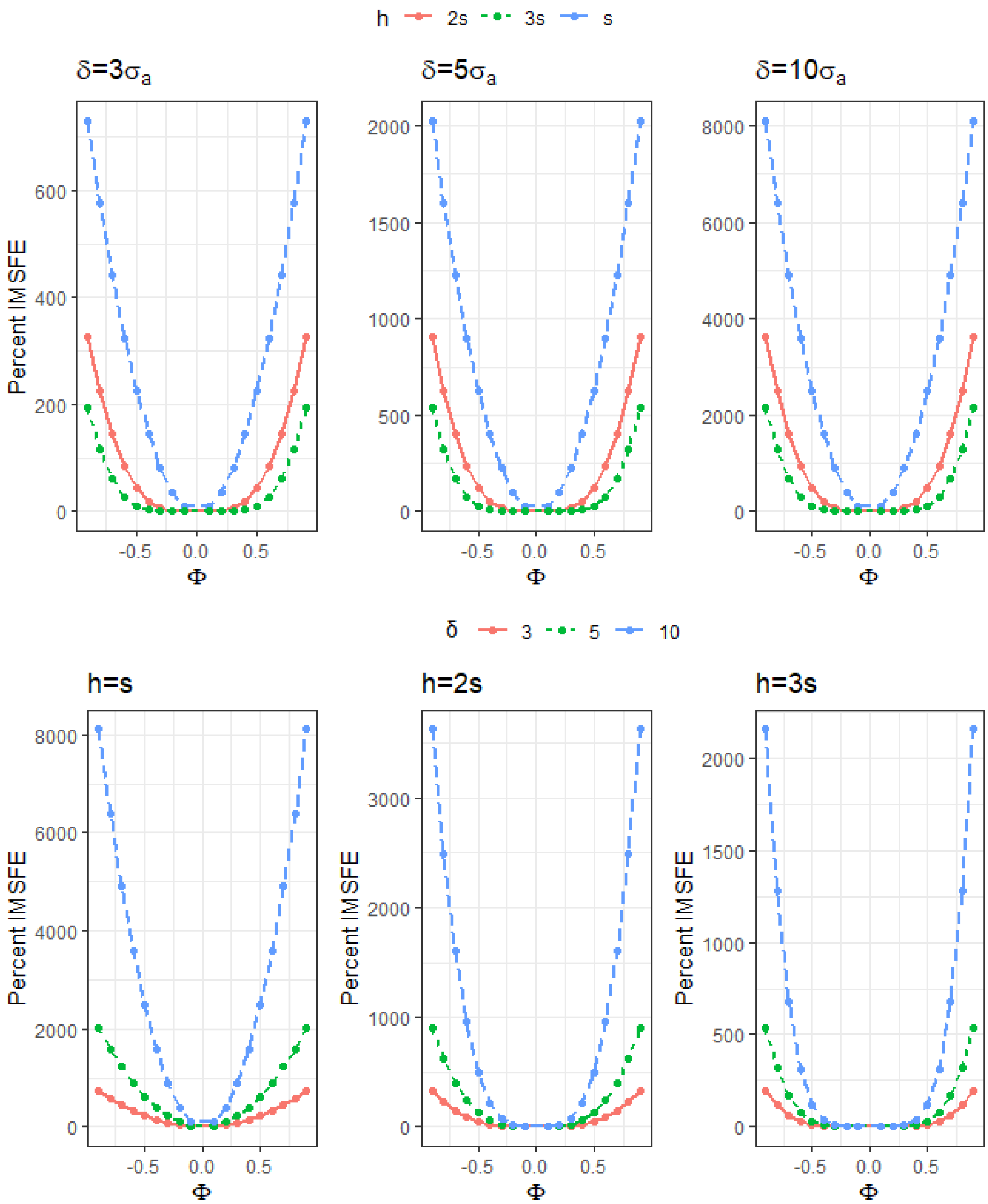

Figure B1.: Percent IMSFE for SAR(1) Model

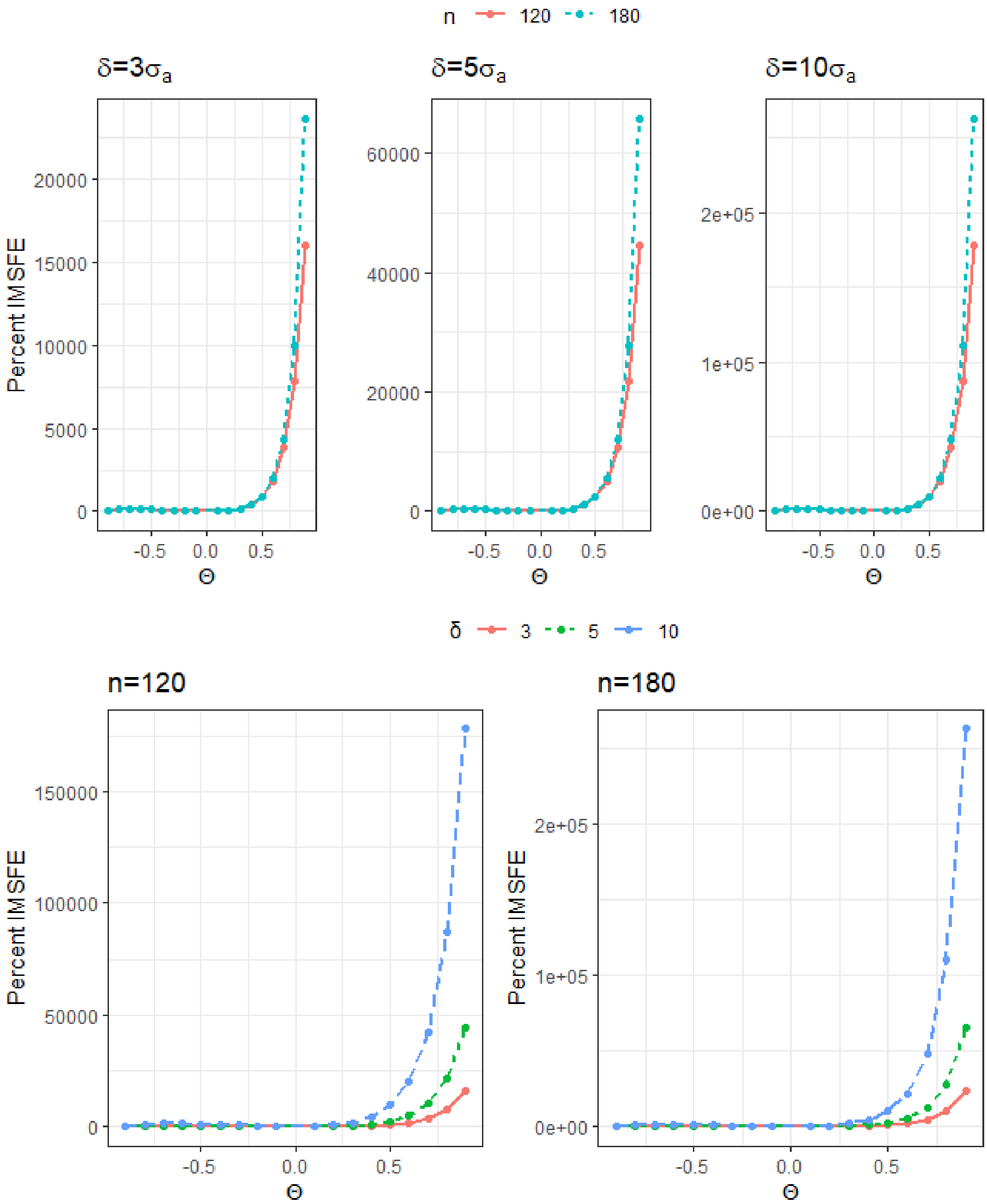

Figure B2.: Percent IMSFE for SMA(1) Model

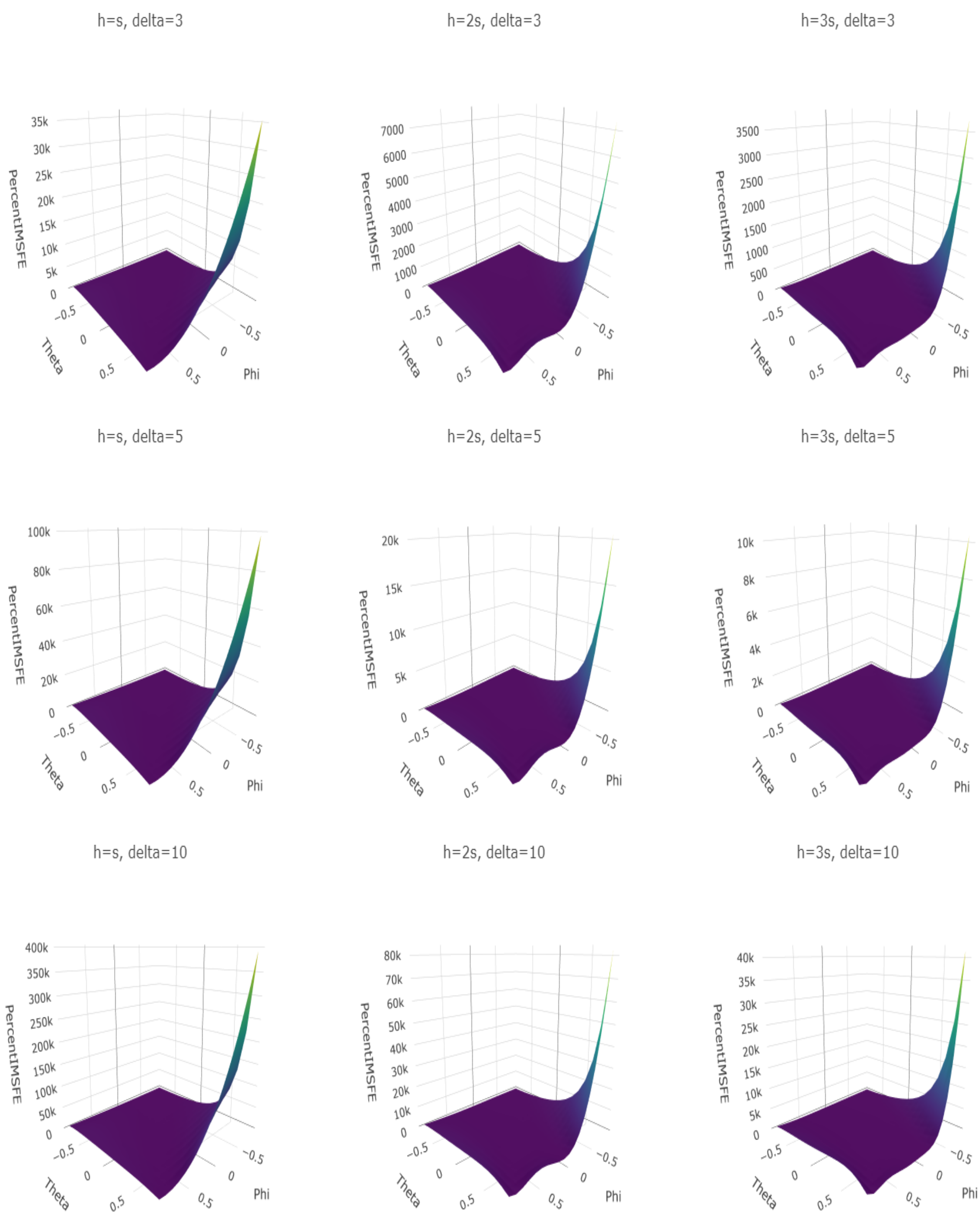

Figure B3.: Percent IMSFE for SARMA(1, 1) Model(n = 120)

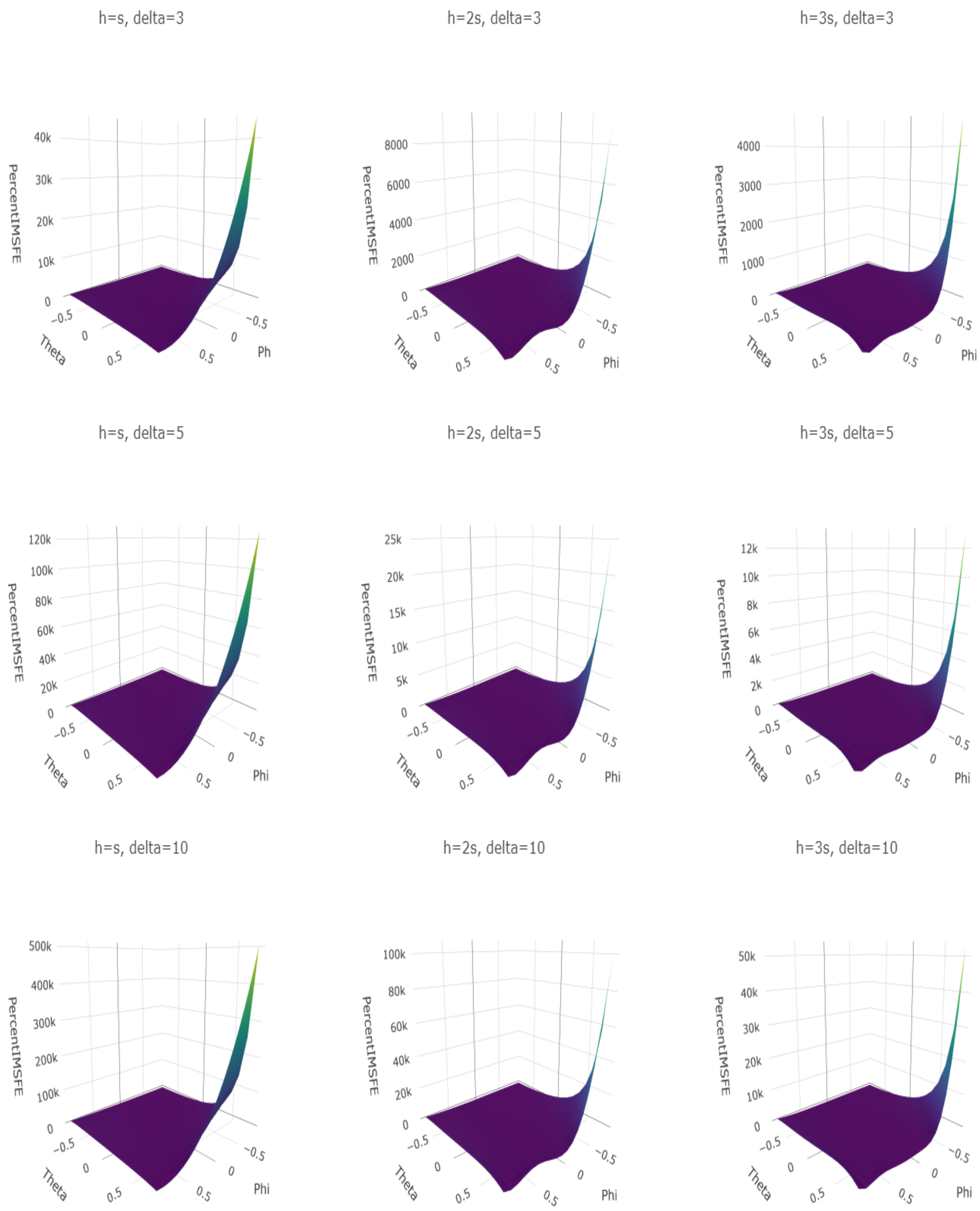

Figure B4.: Percent IMSFE for SARMA(1, 1) Model(n = 180)